185B Homework 2

Due Friday April 6

Question 1 Compute

$$I = \int_0^\infty \frac{dx \sin x}{x^{3/2}} \,.$$

Question 2 Use the Laplace transform and the convolution theorem to derive an integral representation of the Beta-function

$$\beta(m,n) \equiv \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)} \,.$$

These are a continuum generalization of the binomial coefficients.

Question 3 State and prove Jordan's Lemma. (Feel free to concentrate on just one of the four cases.)

Question 4 Consider the contour Γ around the boundary of $\{re^{i\theta} : 0 \leq \theta \leq 2\pi/p,\ 0 \leq r \leq R\}$. Deduce the integral $\int_0^\infty dx/(1+x^p)$, $1 < p \in \mathbb{N}$, by integrating $1/(1+z^p)$ around Γ.

Question 4 Integrate $\log(z)^3/(1+z+z^2)$ around the Hankel contour to show $\int_0^\infty dx \log(x)^2/(1+x+x^2) = 16\sqrt{3}\,\pi^3/9^2$.

Question 5 Integrate $\exp(-z^2)$ around an appropriate contour to establish to Fresnel integrals

$$\int_0^\infty \cos x^2 dx = \int_0^\infty \sin x^2 = \sqrt{\pi/8} \,.$$