# 185B Homework 3

Due Friday April 20

**Question 1** Compute the first correction to Stirling's approximation

$$n! \overset{n\to\infty}{\sim} \frac{\sqrt{2\pi n}\, n^n}{\exp(n)}\,.$$

**Question 2** Generalize Laplace's method to handle $\int_a^b g(x)\exp(th(x))$ for large $t$ when (i) $h(x)$ is maximal at the endpoint $x = b$, (ii) $h(x)$ is extremal at $x_0$ and $x_1$.

**Question 3** Show that

$$\operatorname{erf}(x) \overset{x\to\infty}{\sim} 1 - \frac{e^{-x^2}}{\sqrt{\pi}\, x}\Big\{1 + \sum_{n=0}^{\infty} \frac{(-)^n (2n-1)!!}{2^n x^{2n}}\Big\}\,.$$

**Question 4** Use a contour shaped like a piece of pie subtending an angle $45^\circ$ at the origin to establish the Fresnel integrals

$$\int_{-\infty}^{\infty} e^{\pm iy^2} dy = \sqrt{\pi} e^{\pm i\pi/4}\,.$$

**Question 5** One definition of the Bessel function is as the coefficients in the generating function

$$e^{\frac{z}{2}(w-\frac{1}{w})} = \sum_{n=-\infty}^{\infty} J_n(z) w^n\,.$$

Use this fact to derive an integral representation of the Bessel function.

**Question 6** Re-examine the integral $I = \int_0^\infty \frac{dte^{-t}}{1+xt}$. Compute its asymptotic series valid at small $x$ using the method of integrations by parts. Compare to the result obtained in class. Challenge: compute an asymptotic series valid for large $x$.