

185B Homework 4

Due Friday April 27

Question 1 Study the large t asymptotics of

$$I = \int_{\gamma} \exp[t(x^2 + x^4)]$$

for γ some path in \mathbb{C} .

Question 2 *Airy's Equation:* In class we studied $d^2 f/dx^2 + xf = 0$. Make the variable change $k = x^{1/2}z$ in the Fourier integral and use stationary phase to obtain a second solution

$$f(x) \stackrel{x \rightarrow \infty}{\sim} \frac{B}{x^{1/4}} \cos\left(\frac{2x^{3/2}}{3} - \frac{\pi}{4}\right).$$

Now put $k = -iz|x|^{1/2}$ and use steepest descents to find a solution valid for $x \rightarrow -\infty$.

Question 3 Let $f(z)$ be analytic in some disc radius R about z_0 , in other words $f(z) = \sum_{n=0}^{\infty} a_n(z - z_0)^n$. Suppose $|z_1 - z_0| < R$ so that $f(z) = \sum_{n=0}^{\infty} b_n(z - z_1)^n$. Show that

$$b_n = \sum_{m=0}^{\infty} c_{nm} a_m.$$

(Include an explicit expression for c_{mn} in your argument.)