

## 185B Homework 4

Due Friday April 27

**Question 1** Study the large  $t$  asymptotics of

$$I = \int_{\gamma} \exp[t(x^2 + x^4)]$$

for  $\gamma$  some path in  $\mathbb{C}$ .

**Question 2** *Airy's Equation:* In class we studied  $d^2f/dx^2 + xf = 0$ . Make the variable change  $k = x^{1/2}z$  in the Fourier integral and use stationary phase to obtain a second solution

$$f(x) \xrightarrow{x \rightarrow \infty} \frac{B}{x^{1/4}} \cos\left(\frac{2x^{3/2}}{3} - \frac{\pi}{4}\right).$$

Now put  $k = -iz|x|^{1/2}$  and use steepest descents to find a solution valid for  $x \rightarrow -\infty$ .

**Question 3** Let  $f(z)$  be analytic in some disc radius  $R$  about  $z_0$ , in other words  $f(z) = \sum_{n=0}^{\infty} a_n(z - z_0)^n$ . Suppose  $|z_1 - z_0| < R$  so that  $f(z) = \sum_{n=0}^{\infty} b_n(z - z_1)^n$ . Show that

$$b_n = \sum_{m=0}^{\infty} c_{nm} a_m.$$

(Include an explicit expression for  $c_{mn}$  in your argument.)