185B Homework 7

Question 1 Show that the Riemann Zeta function can be expressed as a product over primes p

$$\zeta(s) = \prod_p \frac{1}{1 - \frac{1}{p^s}}.$$

Question 2 Analytically continue $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$ (valid for $\mathrm{Re}(s) > 1$) to all of $\mathbb{C}$ using the Hankel contour method.

Question 3 Compute

$$\lim_{s \to 1} \left(\zeta(s) - \frac{1}{s-1} \right).$$

Question 4 How many roots does $z + 3 + 2e^z$ have in the left half of the complex plane ($\mathrm{Re}(z) < 0$).