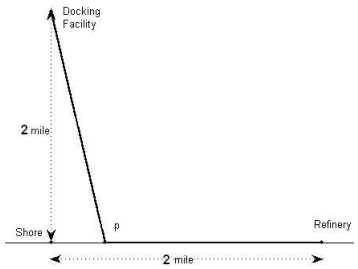
# Math 21A - Final Review

**Question 1** *(20 points)*

Supertankers off-load oil at a docking facility 2 miles offshore. The refinery is 2 miles down the coast from the point on the shore nearest the docking facility. A pipeline must be constructed connecting the docking facility with the refinery. The pipeline costs $ 2,000,000 per mile if constructed underwater and $ 1,000,000 per mile if on land. Find the minimum cost of constructing the pipeline.



**Question 2** *(20 points)*

(i) Write out in words what the following mathematical symbols mean and then give a precise definition:

$$\lim_{x\to\infty} f(x) = L\,.$$

(ii) Use your definition in (i) to *prove*

$$\lim_{x\to\infty} \frac{x^2 - 2x + 2}{x^2 - 2x + 1} = 1\,.$$

**Question 3** *(20 points)* Fill in the blanks

$$\frac{d \log x^2}{dx} = \underline{\qquad\qquad\qquad}$$

$$\frac{d^2 \log x^2}{dx^2} = \underline{\qquad\qquad\qquad}$$

$$\frac{d^3 \log x^2}{dx^3} = \underline{\qquad\qquad\qquad}$$

$$\frac{d^n \log x^2}{dx^n} = \underline{\qquad\qquad\qquad}$$

Explain (but do not prove) why your last answer is correct. (Full points for a well written, concise and logical answer.)

**Question 4** *(20 points)* Find the derivative of: $y = \sin\left(\frac{2}{x} + \frac{5}{x^3}\right)$

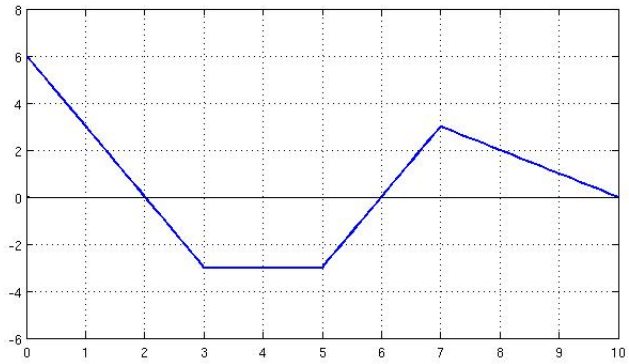
Find $\frac{d}{dx}\sqrt{1 + tan(f(x))} =$

Find $\frac{d^2}{dx^2}\frac{5}{\sqrt{1-x^3}} =$

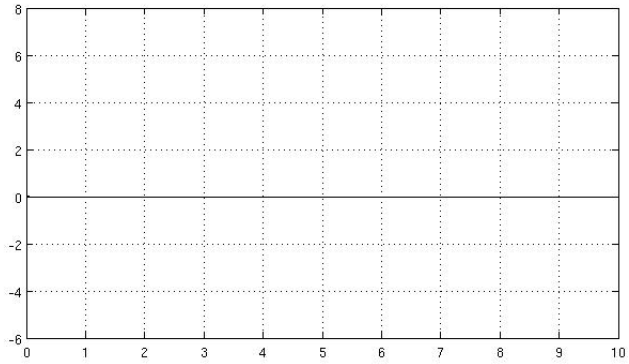
Find $g'(t)$ if $g(t) = e^{9t}\sin^5 2t$

**Question 5** *(20 points)*

Below is the graph of the velicity, in feet per second, of a partical at time $t$ seconds.




Using the graph of the velocity above plot the acceleration of the partical at time $t$.




Given that the partical starts at zero at $t = 0$ and ten seconds later is at a position three feet below the origin, that is $p(0) = 0$ and $p(10) = -3$. Graph the position of the partical verses time below.