

Math 21A: Quiz 4

(Waldron 3/6/08)

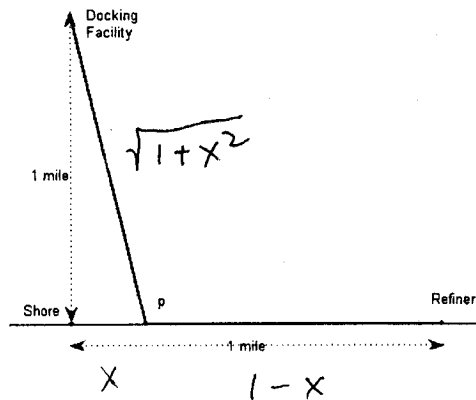
Name : Solutions

Score : 20

This is an open book, closed notes, NO CALCULATOR quiz. Show all work.

Note : $\sqrt{1} = 1$, $\sqrt{2} \approx 1.41$, $\sqrt{3} \approx 1.73$, $\sqrt{4} = 2$, $\sqrt{5} \approx 2.24$

Supertankers off-load oil at a docking facility 1 mile offshore. The refinery is 1 mile down the coast from the point on the shore nearest the docking facility. A pipeline must be constructed connecting the docking facility with the refinery. The pipeline costs \$ 2,000,000 per mile if constructed underwater and \$ 1,000,000 per mile if on land. Find the minimum cost of constructing the pipeline.



$$(5) \text{ Cost}(x) = 2(\sqrt{1+x^2}) + 1(1-x) \text{ in millions of dollars}$$

$$(5) \text{ Cost}(0) = 2 + 1 = 3 \text{ million for straight to shore 1 mile and 1 mile on land.}$$

$$\text{Cost}(1) = 2\sqrt{2} = 2(1.41) = 2.82 \text{ for all water pipeline million}$$

$$\frac{d \text{ Cost}(x)}{dx} = \frac{2x}{\sqrt{1+x^2}} - 1 = 0 \Rightarrow 2x = \sqrt{1+x^2}$$

$$4x^2 = 1+x^2$$

$$x^2 = \frac{1}{3}$$

$$x = \pm \frac{1}{\sqrt{3}} \Rightarrow (5) \quad x = \frac{1}{\sqrt{3}}$$

$$(5) \text{ Cost}\left(\frac{1}{\sqrt{3}}\right) = 2\left(\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2}\right) + 1 - \frac{1}{\sqrt{3}}$$

$$= 2\sqrt{\frac{4}{3}} + 1 - \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}} + 1 - \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} + 1 = 1 + \sqrt{3} = 1 + 1.73$$

$$\text{minimum cost} = \min(3, 2.82, 2.73) = \boxed{2.73 \text{ million dollars}} = 2.73$$