

Q1_____ Last Initial C
 Q2_____ Section A01
 Q3_____ FULL Name CAPTAIN CONUNDRUM
 Q4_____ Student ID # 1.414213562... = $\sqrt{2}$
 Σ _____

A01	18:10-19:00	CRUESS 107	Yan Zhang
A02	19:10-20:00	HART 1130	Blake Hunter
A03	18:10-19:00	HART 1130	Gus Wiseman
A04	19:10-20:00	CRUESS 107	Creed Erickson IV
A05	17:10-18:00	CRUESS 107	James Forehand

MIDTERM EXAMINATION II

21A §A01-05, 13:00-13:50 am

Wednesday February 27, 2008

Declaration of honesty: I, the undersigned, do hereby swear to uphold the highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and unaided by notes, books, calculators, mobile phones, immobile phones or any other electronic device.

Signature C Date 8/27/2

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Question 1

Find the derivatives of the following functions:

$$y = \sin(3x+1) \Rightarrow \frac{dy}{dx} = \frac{d\sin(3x+1)}{d(3x+1)} \frac{d(3x+1)}{dx}$$

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$$= 3 \cos(3x+1)$$

$$y = \frac{1}{(1-\frac{x}{7})^7} \Rightarrow \frac{dy}{dx} = \frac{d(1-\frac{x}{7})^{-7}}{dx} = (-\frac{1}{7})(-7)(1-\frac{x}{7})^{-8}$$

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$$= \frac{1}{(1-\frac{x}{7})^8}$$

$$y = \frac{\tan x}{\tan x + 1} \Rightarrow \frac{dy}{dx} = \frac{d(\frac{\tan x}{\tan x + 1})}{d\tan x} \frac{d\tan x}{dx} = \left(\frac{1}{\tan x + 1} - \frac{\tan x}{(\tan x + 1)^2} \right) \sec^2 x$$

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$$\frac{\cancel{\tan x}(\tan x + 1) - \tan x}{(\sin x + \cos x)^2} = \frac{1}{2\cos x \sin x + 1}$$

$$y = \log(\tan^{-1} x) \Rightarrow \frac{dy}{dx} = \frac{d\log \tan^{-1} x}{dx} \frac{d\tan^{-1} x}{dx}$$

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$$= \frac{1}{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

OR

$$\frac{1}{(\sin x + \cos x)^2}$$

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Question 2

Fill in the blank

$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$$

3

Write the precise definition of the above symbols:

$\forall B < 0 \quad \exists \delta > 0$ such that

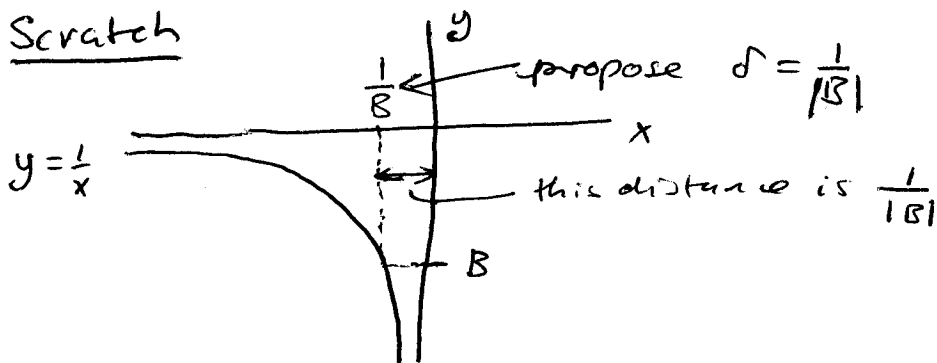
$$x_0 - \delta < x < x_0 \Rightarrow f(x) < B.$$

7

(Here $f(x) = \frac{1}{x}$, $x_0 = 0$)

Prove that your answer for the above limit is correct using this definition.

Scratch



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Proof Let $B < 0$.

Suppose

$$-\frac{1}{|B|} < x < 0$$

$$\Leftrightarrow \begin{matrix} (B \text{ is negative}) \\ \frac{1}{B} < x < 0 \end{matrix}$$

$$\Leftrightarrow B > \frac{1}{x} > 0$$

$$\Rightarrow \frac{1}{x} < B \quad \underline{\underline{Q.E.D.}}$$

$$\frac{25}{25}$$

Question 3

Fill in the blanks

$$\frac{d \log x}{dx} = \frac{1}{x}$$

3

$$\frac{d^2 \log x}{dx^2} = -\frac{1}{x^2}$$

3

$$\frac{d^3 \log x}{dx^3} = +\frac{2}{x^3}$$

2

$$\frac{d^n \log x}{dx^n} = \frac{-(-1)^n (n-1)! x^{-n}}{1}$$

10

Explain (but do not prove) why your last answer is correct. (Full points for a well written, concise and logical answer.)

When you differentiate $\frac{1}{x^m}$

you get $-m \frac{1}{x^{m+1}}$ so each

derivative introduces

(i) a factor $-1 \Rightarrow -(-1)^n$

(ii) a factor $m \Rightarrow (n-1)!$

(iii) a factor $\frac{1}{x} \Rightarrow x^{-n}$

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A rigorous proof using induction is not difficult.

Be open minded when * is correct, otherwise give few points

$$\frac{25}{25}$$

Question 4

Derive a formula relating the logarithmic derivative

$$\frac{d \log f(x)}{dx}$$

to the derivative of $f(x)$. Use your formula to compute

$$\frac{d \left(\frac{(x-1)(x-2)^2(x-3)^3}{(x+1)(x+2)^2(x+3)^3} \right)}{dx}$$

(answers expressed as a sum of fractions will be accepted).

$$\frac{d \log f(x)}{dx} = \frac{d \log f(x)}{df(x)} \frac{df}{dx} = \frac{1}{f(x)} \frac{df}{dx}$$

10 $\Rightarrow$ $\boxed{\frac{df}{dx} = f \frac{d \log f}{dx}}$

Hence compute

$$\begin{aligned} \frac{d \log \left(\frac{(x-1)(x-2)^2(x-3)^3}{(x+1)(x+2)^2(x+3)^3} \right)}{dx} &= \frac{d}{dx} \left(\log(x-1) + 2 \log(x-2) + 3 \log(x-3) \right. \\ &\quad \left. - \log(x+1) - 2 \log(x+2) - 3 \log(x+3) \right) \\ &= \frac{1}{x-1} + \frac{2}{x-2} + \frac{3}{x-3} - \frac{1}{x+1} - \frac{2}{x+2} - \frac{3}{x+3} \end{aligned}$$

$$\begin{aligned} \Rightarrow \frac{d}{dx} \left(\frac{(x-1)(x-2)^2(x-3)^3}{(x+1)(x+2)^2(x+3)^3} \right) &= \frac{(x-2)^2(x-3)^3}{(x+1)(x+2)^2(x+3)^3} + \frac{2(x-1)(x-2)(x-3)^3}{(x+1)(x+2)^2(x+3)^3} + \frac{3(x-1)(x-2)^2(x-3)^2}{(x+1)(x+2)^2(x+3)^3} \\ &\quad - \frac{(x-1)(x-2)^2(x-3)^3}{(x+1)^2(x+2)^2(x+3)^3} - \frac{2(x+1)(x-2)^2(x-3)^3}{(x+1)(x+2)^3(x+3)^3} - \frac{3(x-1)(x-2)^2(x-3)^3}{(x+1)(x+2)^2(x+3)^4} \end{aligned}$$

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