Math 21A: Quiz 1
Section 2 (7:10-8:00)
(Waldron 1/17/08)

Name : Solutions

This is a closed book, no calculator quiz.

1. (5 points) Define: The limit of a function $f(x)$ at a point x_0.

For all $\varepsilon > 0$ there exists a $\delta > 0$ such that if $|x - x_0| < \delta$ then $|f(x) - L| < \varepsilon$.

2. (15 points) Prove: The limit of $f(x) = \frac{x}{2} - 3$ at $x_0 = 5$ is $-\frac{1}{2}$.

side work:

$|f(x) - L| < \varepsilon$

$\left| \frac{x}{2} - 3 - (-\frac{1}{2}) \right| < \varepsilon$

$-\varepsilon < \frac{x}{2} - \frac{5}{2} < \varepsilon$

$-2\varepsilon < x - 5 < 2\varepsilon$

$|x - 5| < 2\varepsilon$

let $\delta = 2\varepsilon$

Proof:

Let $\varepsilon > 0$ and $\delta = 2\varepsilon$.

Assume $|x - 5| < \delta$

$\Rightarrow |x - 5| < 2\varepsilon$

$\Rightarrow \left| \frac{x}{2} - \frac{5}{2} \right| < \varepsilon$

$\Rightarrow \left| \frac{x}{2} - 3 - (-\frac{1}{2}) \right| < \varepsilon$

$\Rightarrow \left| f(x) - (-\frac{1}{2}) \right| < \varepsilon$

Thus $\lim\limits_{x \to 5} \frac{x}{2} - 3 = -\frac{1}{2}$ ■