Q1__________ **Last Initial** ____________________________________

Q2__________ **Section** ____________________________________

(A01 Andy Port, A02 Jacob Porter, A03 Josh Clement, A04 Brandon Barrette)

Q3__________

Q4__________ **FULL Name** ____________________________________

Q5__________

Q6__________

Σ __________

SAMPLE FINAL EXAMINATION

21D 2009

Declaration of honesty: You are paying the University a lot of money for us to give you an honest evaluation of your abilities. Cheating is a waste of your money and our time. Sign below if you agree.

Signature ____________________ **Date** _______________

Question 1 The base of a pile of sand forms a disc of radius R. The height of the pile above any point in the base is proportional to the square root of the distance of that point from the edge of the disc with constant of proportionality λ. Write an expression for the volume of the sandpile as (i) a double integral and (ii) a triple integral. Using either expression, compute the volume of the sandpile.

Question 2 Sketch the region of integration and evaluate the integral

$$\int_0^3 \int_0^2 (4 - y^2) dy\, dx \,.$$

Question 3 A thin, constant density plate covers the region in the xy-plane bounded by the ellipse $(x/a)^2 + (y/b)^2 = 1$ (where $a, b > 0$). Find the polar moment of inertia of the plate about the origin.

Question 4 Captain Conundrum carefully pours an incompressible fluid onto the xy-plane at the origin. At all times he pours at a constant rate and tries to keep the fluid level constant. Moreover, the fluid flows radially outward from the origin. (i) Write down a two-dimensional vector field that would model the rate and direction at which fluid passes any point. (ii) Compute the divergence of this vector field. (iii) Use Green's theorem to explain why the flux leaving any region that does not contain the origin must vanish. (iv) Is the same true for regions containing the origin? Explain.

Question 5 Find the counterclockwise circulation and outward flux for the vector field

$$\vec{F} = (2xy + x)\mathbf{i} + (xy - y)\mathbf{j} \,,$$

and the curve C bounding the unit square with one corner at the origin of the first quadrant of the xy-plane.

Question 6 Show that the volume V of a region D in space enclosed by the oriented surface S with outward unit normal $\vec{n}$ satisfies the identity

$$V = \frac{1}{3} \int_S \vec{r} \cdot \vec{n} d\sigma \,,$$

where $\vec{r}$ is the position vector of the point (x, y, z) in D.