Solution to Quiz 2

15.6 #14

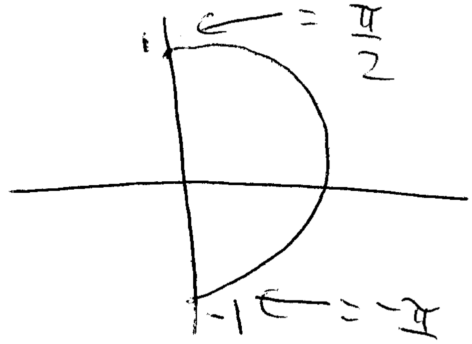
$$\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{0}^{x}(x^2+y^2)\,dz\,dx\,dy=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0}^{1}\int_{0}^{r\cos\theta} r^3\,dz\,dr\,d\theta$$

$$x^2+y^2=r^2\cos^2\theta+r^2\sin^2\theta$$
$$=r^2$$
$$r^2\cdot dz\,dx\,dy$$
$$=r^2\cdot dz\cdot r\cdot dr\cdot d\theta$$
$$=r^3\,dz\,dr\,d\theta$$

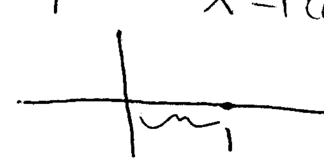
$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{0}^{1} r^4\cos\theta\,dr\,d\theta=\frac{1}{5}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\,d\theta=\boxed{\frac{2}{5}}$$

Explanation

-1 to 1
$= -\frac{\pi}{2}$ to $\frac{\pi}{2}$ because



0 to $\sqrt{1-y^2}$
$= 0$ to 1 because $x^2+y^2=1 \Rightarrow \sqrt{1-y^2}=x=r\cos\theta$



0 to x
$= 0$ to $r\cos\theta$ because $x=r\cos\theta$ in polar coordinate