

Quiz 4: Solution

May 19

16.4 #14) Find the counterclockwise circulation of $\vec{F} = (y + e^x \ln y)\mathbf{i} + (e^x/y)\mathbf{j}$ around the boundary of the region that is bounded above by the curve $y = 3 - x^2$ and below by the curve $y = x^4 + 1$.

Solution

$$M = y + e^x \ln y, \quad N = \frac{e^x}{y} \Rightarrow \frac{\partial M}{\partial y} = 1 + \frac{e^x}{y}, \quad \frac{\partial N}{\partial x} = \frac{e^x}{y}$$

Counter-clockwise

$$\Rightarrow \text{Circulation} = \iint_R \left[\frac{e^x}{y} - \left(1 + \frac{e^x}{y}\right) \right] dx dy$$

$$= \cancel{\iint_R (-1) dx dy} = \iint_R (-1) dx dy = \int_{-1}^1 \int_{x^4+1}^{3-x^2} -1 dy dx$$

$$= - \int_{-1}^1 [(3 - x^2) - (x^4 + 1)] dx$$

$$= \int_{-1}^1 (x^4 + x^2 - 2) dx$$

$$= \left[\frac{x^5}{5} + \frac{x^3}{3} - 2x \right]_{-1}^1$$

$$= \frac{1}{5} + \frac{1}{3} - 2 - \left(-\frac{1}{5} - \frac{1}{3} + 2 \right)$$

$$= \boxed{-\frac{44}{15} = \text{Final Answer}}$$