# 265 Take Home Final

Due Friday December 16, 1pm

**Question 1**
Compute the first relativistic correction to the Hamiltonian for the Hydrogen atom implied by the Dirac equation. (*Hint:* you should find 3 terms – a kinetic energy correction, a spin orbit coupling and a point-like Darwin term). Bonus: calculate the shift of the ground state energy and compare your result with experiment.

**Question 2**
Quantize and solve the motion of a charged particle in a constant magnetic field.

**Question 3**
Consider the metric ansatz

$$ds^2 = -\Delta(r)dt^2 + \frac{N(r)^2 dr^2 + R(r)^2(d\theta^2 + \sin^2\theta d\phi^2)}{\Delta(r)} .$$

Show that the Einstein–Hilbert action becomes

$$S = 4\pi \int dr \left[ \frac{1}{N(r)}\left( -R'(r)^2 + \frac{R^2\Delta'(r)^2}{4\Delta(r)^2} \right) - N(r) \right] .$$

Calculate the resulting equations of motion and verify that the Schwarzschild black hole is a solution. Discuss the quantization of this system...