Homework 2

The Lie bracket.

Q1 Suppose v, w are vector fields.
Is it always possible (at least locally) to find coordinates $x^\mu = (x, y, \dots)$ such that

$$v = \frac{\partial}{\partial x} \quad , \quad w = \frac{\partial}{\partial y} \quad ?$$

Q2 Consider the sphere S^2 with spherical coordinates (θ, φ) again. Here are three interesting vector fields

$$\begin{cases} \dfrac{\partial}{\partial \varphi} \\ \sin\varphi \dfrac{\partial}{\partial \theta} + \cot\theta \cos\varphi \dfrac{\partial}{\partial \varphi} \\ \cos\varphi \dfrac{\partial}{\partial \theta} - \cot\theta \sin\varphi \dfrac{\partial}{\partial \varphi} \end{cases}$$

Since vectors are linear differential operators it is possible to compute their commutators (i.e. $uv - vu \equiv [u, v]$) or "Lie Brackets".

(i) interpret the above vector fields

(ii) compute their Lie brackets (comment on your result).

(iii) Justify or re-evaluate your answer to Q1 using (ii).