

Homework #4

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Let $| \pm \rangle$ be an orthonormal basis for a 2-state system. Let

$$S_1 = \frac{1}{2} \{ |+\rangle\langle -| + |-\rangle\langle +| \}$$

$$S_2 = -\frac{i}{2} \{ |+\rangle\langle -| - |-\rangle\langle +| \}$$

$$S_3 = \frac{1}{2} \{ |+\rangle\langle +| - |-\rangle\langle -| \}$$

① verify

$$S_i S_j - S_j S_i = i \epsilon_{ijk} S_k$$

where ϵ_{ijk} is totally antisymmetric and $\epsilon_{123} \equiv 1$, as well as

$$S_i S_j + S_j S_i = \frac{1}{2} \delta_{ij}$$

② Suppose the system is known to be in an eigenstate of $\vec{S} \cdot \vec{n}$

where $\vec{S} = (S_1, S_2, S_3)$ and $\vec{n}$

is a ~~eigenstate~~ unit vector.

What is the probability of observing $\frac{1}{2}$ when S_x is measured. Explain why your answer depends how it does on $\vec{n}$.