

Homework 6

Differential forms

Introduce anticommuting quantum oscillators $b_\mu, b^{+\mu}$ obeying

$$\{b_\mu, b^{\nu+}\} = b_\mu b^{\nu+} + b^{\nu+} b_\mu = \delta_\mu^\nu$$

$$\{b_\mu, b_\nu\} = 0 = \{b^{+\mu}, b^{+\nu}\}$$

and

$$b_\mu |0\rangle = 0$$

Mimic the symmetric form technology introduced in class as far as you can.

In particular

- ① Explain the relationships between states and differential forms.

② Introduce an appropriate covariant derivative operator ∇_μ

③ Use ∇_μ to construct operators

$$d = b^{\dagger\mu} \nabla_\mu$$

$$N = b^{\dagger\mu} b_\mu$$

$$\Delta = \{\delta, d\}$$

$$\delta = b_\mu g^{\mu\nu} \nabla_\nu$$

④ Compute the superalgebra of N, d, δ, Δ . i.e.

$$[N, d], [N, \delta], [N, \Delta],$$

$$d^2, \delta^2, \{d, \delta\}, [d, \Delta], [\delta, \Delta].$$

⑤ Optional: Find the underlying classical system.