265 Take Home 2

Due Last Day of Class

Question 1
Compute infinitesimal generators of the G-action on the minimal coadjoint orbit for $G = Sl(3, \mathbb{R})$. Quantize and interpret this system[1]. [Hints: Warm up on the case $G = Sl(2, \mathbb{R})$. Feel free to use a computer to perform some of the heavier matrix algebra.]

Question 2
Fermionic coherent states: Consider the fermionic oscillator algebra

$$\{b, b^\dagger\} = 1 \, , \quad \{b, b\} = \{b^\dagger, b^\dagger\} = 0.$$

Develop a theory of coherent states for fermions. Study the action

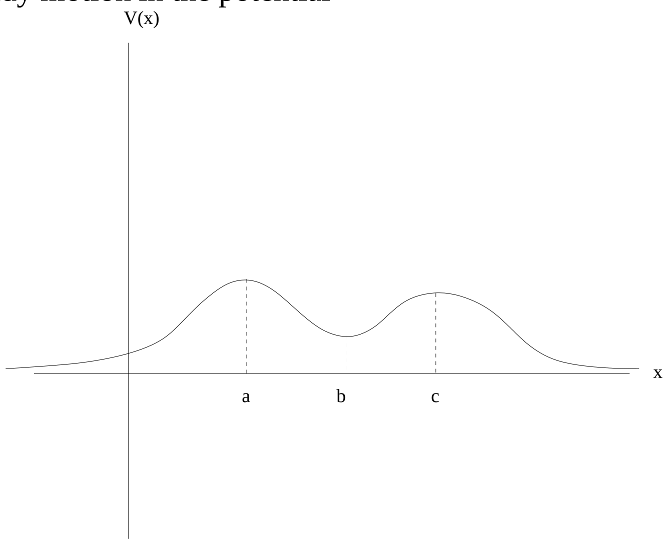
$$S = \int dt \{ b_1 \dot{b}_1 + b_2 \dot{b}_2 + i b_1 b_2 \} \, ,$$

where $b_i(t)$ are Grassmann variables. Quantize the system and compute its coherent state propagator. Can you represent this as a path integral?

Question 3 What is a squeezed state? What are its applications?

[1] Hint: In the conventions employed in class you should find something like $e_{-\omega} = \hbar\pi^2 + pq\pi$. Study the symplectomorphism $\hbar = R^2/2$, $q = Rx/2$, $\pi = P/R - x\Pi/R^2$ and $p = 2\Pi/R$.

Question 4
Study motion in the potential



semiclassically using path integral methods.

Question 5 *Slightly tangential....* Compute an asymptotic series expansion for $I(x)$ valid for small x where

$$I(x) = \int_0^\infty \frac{e^{-t}dt}{1+xt}\,.$$

[Hint: use a geometric series for the denominator.] Is your power series convergent? Include an estimate for the error made by terminating your expansion after N terms. Use a computer to numerically evaluate $I(x)$ for various small values of x. Compare these values to estimates obtained from the asymptotic series. Is $I(x)$ analytic at $x = 0$?