

## Determinants

Determinants are a great tool so long as you know the main tricks for manipulating them:

*Give the definition* of the determinant of an  $n \times n$  matrix  $M$ . If matrices  $M$  and  $M'$  are related by

- (1)  $M'$  is the transpose of  $M$ ,
- (2)  $M$  is the same as  $M'$  except two rows have been swapped,
- (3)  $M$  is the same as  $M'$  except two columns have been swapped,
- (4)  $M$  is the same as  $M'$  except  $\lambda$  times the  $k$ th row has been added to the  $j$ th row,
- (5)  $M$  is the same as  $M$  except one row has been multiplied by  $\lambda$ ,
- (6)  $M' = \lambda M$ ,

then for each case *state the relationship* between  $\det M$  and  $\det M'$ . Also *give a proof* of the relationship you have stated.

Now let  $M = I$ , the identity matrix and let  $N$  be a  $n \times m$  matrix. For each of the above cases calculate the matrix  $M'$  and explain how the matrix  $M'N$  is related to  $N$ .