

Determinants

Determinants are a great tool so long as you know the main tricks for manipulating them:

Give the definition of the determinant of an $n \times n$ matrix M . If matrices M and M' are related by

- (1) M' is the transpose of M ,
- (2) M is the same as M' except two rows have been swapped,
- (3) M is the same as M' except two columns have been swapped,
- (4) M is the same as M' except λ times the k th row has been added to the j th row,
- (5) M is the same as M' except one row has been multiplied by λ ,
- (6) $M' = \lambda M$,

then for each case *state the relationship* between $\det M$ and $\det M'$. Also *give a proof* of the relationship you have stated.

Now let $M = I$, the identity matrix and let N be a $n \times m$ matrix. For each of the above cases calculate the matrix M' and explain how the matrix $M'N$ is related to N .