Last Initial ______________________________

FULL Name ______________________________

Student ID ______________________________

67 MIDTERM I

3:10pm-4:00pm, Wednesday February 1, 2012

Declaration of honesty: I, the undersigned, do hereby swear to uphold the highest standards of academic honesty, including, but not limited to, submitting work that is original, my own and unaided by books, calculators, pet rocks, the secret service, computers, mobile phones, immobile phones, muses, ruses, calculus superheros or any other electronic device.

Only carefully set out work is guaranteed full credit. Focus on doing questions well rather than scoring partial credit.

Signature ____________________ **Date** _______________

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Question 1

Let V be a vector space over $\mathbb{F}$. **Give definitions** for the following:

(i) A subspace U of V.

(ii) $\operatorname{span}(v_1, \ldots, v_n)$ where $v_i \in V$ $(i = 1, \ldots, n)$.

Now let $v_1, \ldots, v_n$ and $w_1, \ldots, w_m$ be vectors in V. Moreover suppose

$$w_1, \ldots, w_m \in \operatorname{span}(v_1, \ldots, v_n) \qquad \text{and} \qquad v_1, \ldots, v_n \in \operatorname{span}(w_1, \ldots, w_m)\,.$$

Prove

$$\operatorname{span}(v_1, \ldots, v_n) = \operatorname{span}(w_1, \ldots, w_m)\,.$$

Extra space for question 1.

Question 2

Give the definition of a vector space V over a field $\mathbb{F}$ (*do not define what a field is*):

Prove or disprove:

(i) The set of integers $\mathbb{Z}$ is a vector space over the rationals $\mathbb{Q}$ with standard rules for addition and scalar multiplication.

(ii) The set of integers $\mathbb{Z}$ is a vector space over bits $\mathbb{Z}_2$ with standard rules for addition and scalar multiplication.

...to be continued

(iii) The subset $\{(x, y, z, w) | \, x + 2y - z + w = 0\}$ of the vector space $\mathbb{F}^4$ over $\mathbb{F}$ with addition and scalar multiplication inherited from the standard rules in $\mathbb{F}^4$ is itself a vector space.

(iv) The subset $\{(x, x+y, x-y, y) | \, x, y \in \mathbb{Z}_2\}$ of the vector space $\mathbb{Z}_2^4$ over $\mathbb{Z}_2 = \{0, 1\}$ with addition and scalar multiplication inherited from the standard rules in $\mathbb{Z}_2^4$ is itself a vector space.

Extra space for question 2

Question 3

On the next page, points in the vector space $\mathbb{Z}_3^4 = \{(x, y, z, w)|\, x, y, z, w \in \mathbb{Z}_3\}$ over $\mathbb{Z}_3 = \{0, 1, 2\}$ are depicted as positions in a 4-dimensional game of TIC-TAC-TOE. (The zero vector $(0, 0, 0, 0)$ is at the extreme top left and the vector $(2, 2, 2, 2)$ is at the bottom right.)

(i) Mark on the diagram the vectors

$$(1, 1, 0, 0)\,,\ (0, 0, 2, 2)\,,\ (1, 0, 1, 1)\ \text{and}\ (2, 1, 0, 2)\,.$$

(Hint: To keep things tidy, use the shorthand 1100 and write small!)

(ii) Let $U = \{(x, y, z, z)|\, x, y, z \in \mathbb{Z}_3\}$. Put an O on all locations corresponding to points in U. How many O's were required? Is U a subspace?

(iii) Let $V = \{(x, y, z, w) \in \mathbb{Z}_3^4|\, x + y + z + w = 0, z + w = 0\}$. Put an X on all locations corresponding to points in V. How many X's did you need? Is V a subspace?

(iv) Does

$$U + V = U \oplus V\ ?$$

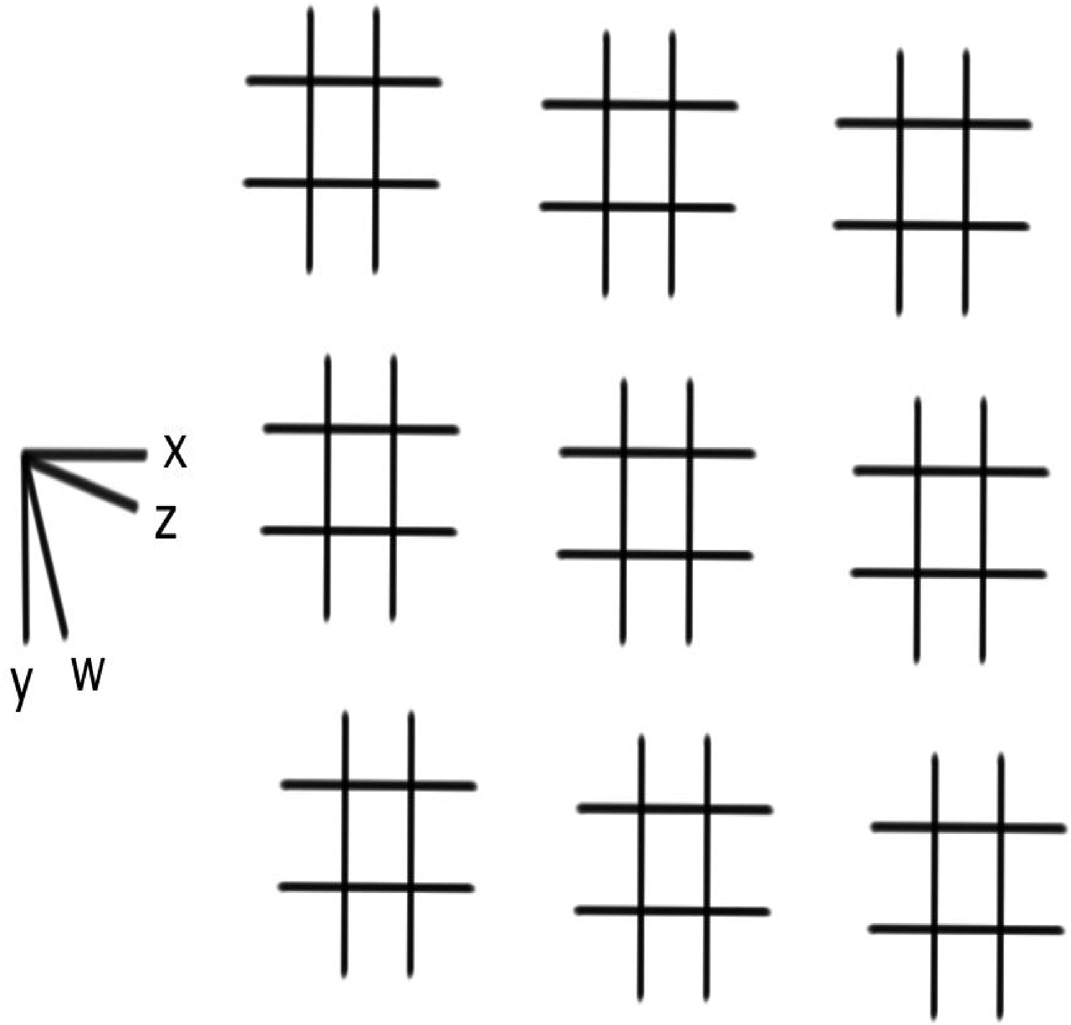
Explain.

(v) Write down in set notation a subspace corresponding to a line L in $\mathbb{Z}_3^4$ such that $L + U = \mathbb{Z}_3^4$. Mark a $\square$ for each point on your chosen line. How many squares did this take?

(vi) Suppose W is a (non-empty) subspace of $\mathbb{Z}_3^4$. Let

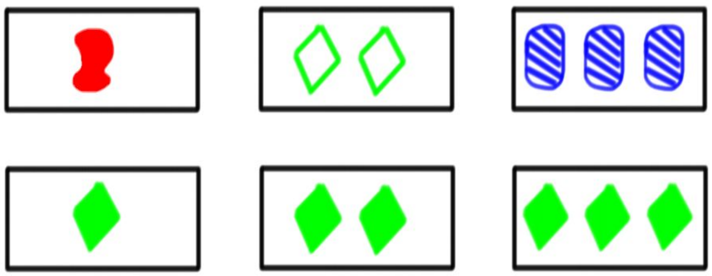
$$w := \log_3 \big(\text{card}\, W\big)\,.$$

(Remember that the cardinality is the number of elements of a set.) List all possible values for w.



To be continued

(vii) *Harder:* In the game Set®, each card has four attributes–number, shape, shading and color for each of which there are three possibilities. Players race one to yell out "set" when they recognize three cards that, for each attribute, have that attribute all the same, or all different. Here is a picture of some set cards; each horizontal line of cards is a "set".



Explain how to identify set cards with points in $\mathbb{Z}_3^4$. Can you write down subspaces corresponding to "sets"?

Extra space for question 3

Question 4 *(Extra Credit)*

Use this space for feedback (positive or negative) that could help improve the course: