Math 21D
Vogler
Discussion Sheet 2

1.) Consider a flat plate lying in the region bounded by the graphs of $y = e^x, x = 0$, and $y = 2$. Assume that density at point (x, y) is given by $\delta(x, y) = x^2y^3 + 1$.

a.) Set up but do not evaluate a double integral which represents the area of the plate.

b.) Set up but do not evaluate a double integral which represents the mass of the plate.

c.) Set up but do not evaluate double integrals which represent the centroid of the plate.

d.) Set up but do not evaluate double integrals which represent the center of mass of the plate.

e.) Set up but do not evaluate double integrals which represent the moment of inertia of the plate about

i.) the origin.

ii.) the x-axis.

iii.) the line $x = 4$.

2.) Consider a flat plate lying in the region bounded by the graphs of $x = y^2$ and $x = 2-y$. Assume that density at point (x, y) is given by $\delta(x, y) = \ln(x^2y^2 + 4)$.

a.) Set up but do not evaluate a double integral which represents the area of the plate.

b.) Set up but do not evaluate a double integral which represents the mass of the plate.

c.) Set up but do not evaluate double integrals which represent the centroid of the plate.

d.) Set up but do not evaluate double integrals which represent the center of mass of the plate.

e.) Set up but do not evaluate double integrals which represent the moment of inertia of the plate about

i.) the origin.

ii.) the y-axis.

iii.) the line $y = -3$.

3.) Consider region R bounded by the graphs of $x = y^3$, $x = 2$, and $y = 0$. Find the

a.) average height of region R.

b.) average width of region R.

c.) average distance from points (x, y) in R to the point $(0, 4)$.

4.) Let R be the region in the first quadrant on or inside the circle $x^2 + y^2 = 9$.

a.) Describe R using vertical cross-sections.

b.) Describe R using horizontal cross-sections.

c.) Describe R using polar coordinates in the format

i.) $a \le \theta \le b,\ f(\theta) \le r \le g(\theta)$
ii.) $a \le r \le b,\ f(r) \le \theta \le g(r)$

5.) Let R be the region bounded by the graphs of $y = x, x = 0$, and $y = 3$.
a.) Describe R using vertical cross-sections.
b.) Describe R using horizontal cross-sections.
c.) Describe R using polar coordinates in the format
i.) $a \le \theta \le b,\ f(\theta) \le r \le g(\theta)$
ii.) $a \le r \le b,\ f(r) \le \theta \le g(r)$

6.) Let R be the region on or inside the circle $x^2 + (y-2)^2 = 4$.
a.) Describe R using vertical cross-sections.
b.) Describe R using horizontal cross-sections.
c.) Describe R using polar coordinates in the format
i.) $a \le \theta \le b,\ f(\theta) \le r \le g(\theta)$
ii.) $a \le r \le b,\ f(r) \le \theta \le g(r)$

7.) Evaluate the following double integrals.

a.) $\displaystyle\int_0^{\pi/2}\int_0^{\sin\theta} r\cos\theta\,dr\,d\theta$ b.) $\displaystyle\int_0^{\pi}\int_0^{1+\cos\theta} r\,dr\,d\theta$

c.) $\displaystyle\int_{-\pi/2}^{\pi/2}\int_0^{\sin\theta} r^2\,dr\,d\theta$ d.) $\displaystyle\int_0^{\pi}\int_0^{1-\sin\theta} r^2\cos\theta\,dr\,d\theta$

8.) For each of the following problems, sketch the two-dimensional region described by the iterated integral, convert to polar coordinates, and evaluate the double integral.

a.) $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}} (x^2+y^2)\,dy\,dx$ b.) $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-(x^2+y^2)}\,dx\,dy$

c.) $\displaystyle\int_0^2\int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\,dy\,dx$ d.) $\displaystyle\int_0^4\int_3^{\sqrt{25-x^2}} dy\,dx$

9.) Use a double integral to find the area of the region in the first quadrant between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

10.) Find the average distance from points (x, y) on or inside a circle of radius r to the center of the circle.

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

11.) The minute and hour hand of a watch line up perfectly at 12 o'clock. In how many minutes and seconds will the hands line up perfectly again ?

Thanks to Dr. Kouba