Math 21D
Vogler
Discussion Sheet 3

1.) Let R be the solid region bounded by the hemi-sphere $z = \sqrt{4 - x^2 - y^2}$ and the plane $z = 0$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid

a.) using rectangular coordinates.

b.) using cylindrical coordinates.

2.) Let R be the solid region bounded by the cone $z = \sqrt{x^2 + y^2}$ and the hemi-sphere $z = \sqrt{18 - x^2 - y^2}$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid

a.) using rectangular coordinates.

b.) using cylindrical coordinates.

3.) Let R be the solid region inside the cylinder $x^2 + y^2 = 4$ and bounded by the plane $z = 0$ and the hemi-sphere $z = \sqrt{9 - x^2 - y^2}$. SET UP BUT DO NOT EVALUATE triple integrals which represent the volume of the solid

a.) using rectangular coordinates.

b.) using cylindrical coordinates.

4.) Let R be the solid region enclosed by the paraboloid $z = 6 - x^2 - y^2$ and the cone $z = \sqrt{x^2 + y^2}$. If the temperature at point $P = (x, y, z)$ is given by $T = \ln(x^2 + y^2 + z^2 + 1)$, then SET UP BUT DO NOT EVALUATE triple integrals representing the average temperature of the solid

a.) using rectangular coordinates.

b.) using cylindrical coordinates.

5.) Consider the UFO bounded by the surfaces $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$. The density of the UFO at point $P = (x, y, z)$ is given by the square of the distance from P to the $z-$axis. SET UP BUT DO NOT EVALUATE triple integrals in rectangular and cylindrical coordinate systems which represent the UFO's

a.) average density.

b.) total mass.

c.) total volume.

d.) $x-$coordinate of the centroid.

e.) $y-$coordinate of the center of mass.

f.) moment of inertia about

i.) the origin.

i.) the z-axis.

ii.) the line parallel to the x-axis and passing throught the point $(3, -4, 5)$.

6.) Convert the following cylindrical integral to rectangular coordinates. DO NOT EVALUATE THE INTEGRAL.

$$\int_0^{2\pi} \int_0^{2} \int_0^{5-r\cos\theta-r\sin\theta} r^4 \sin^2\theta\cos\theta \; dz\, dr\, d\theta$$

7.) Convert the following rectangular integral to cylindrical coordinates. DO NOT EVALUATE THE INTEGRAL.

$$\int_{-2}^{2} \int_{2-\sqrt{4-x^2}}^{2+\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} (x^2+y^2)^3 \cos z \; dz\, dy\, dx$$

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

8.) The camp cook wants to measure exactly four ounces of vinegar out of a jug, but has only a five-ounce container and a three-ounce container. How can the cook accomplish the task ?

Thanks to Dr. Kouba