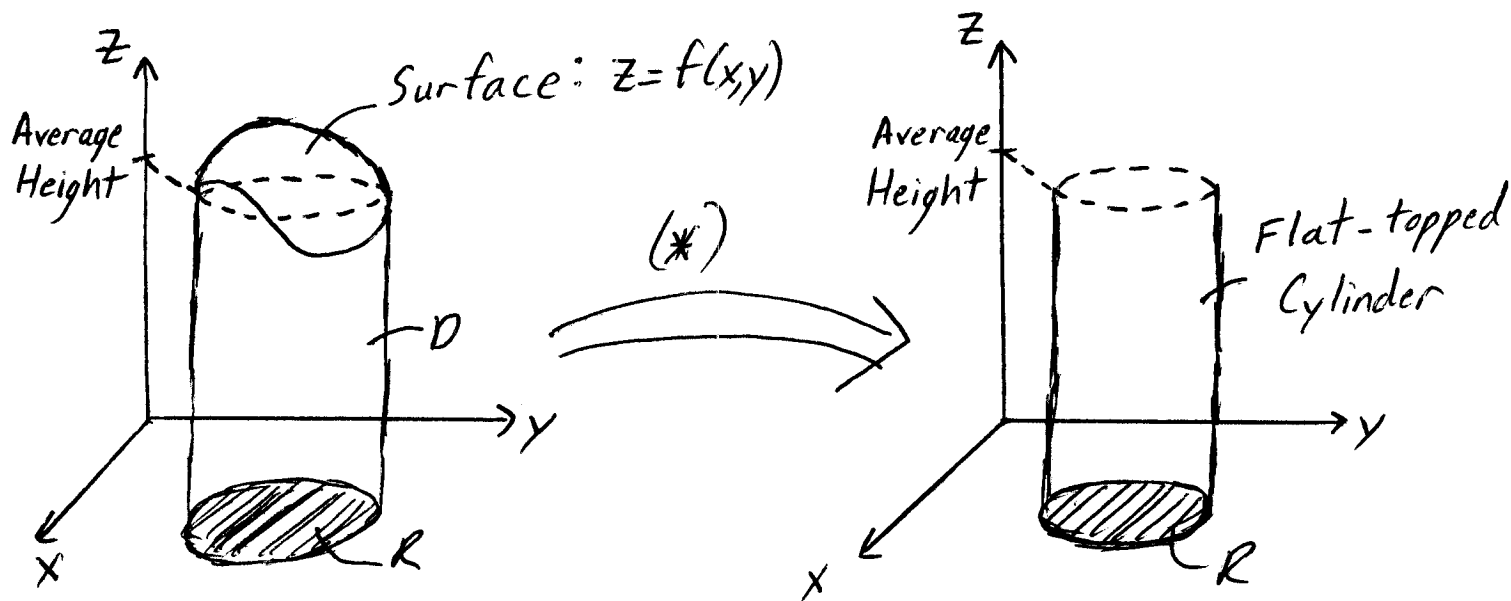


Math 21D
Vogler

Average Value of $f(x,y)$ over 2D region R

Consider a solid D beneath the surface $z = f(x,y)$ (i.e. height function) & above 2D region R .



To find average value/height, we find a flat-topped cylinder whose volume equals that of original solid D

$$\Rightarrow \iint_R f(x,y) dA = \text{Vol. of } D = \text{Vol. of Cylinder} = (\text{Ave. Height})(\text{Area of } R)$$

Algebraic manipulation leads to following:

Defn The Average Value of $f(x,y)$ on region R is

$$\text{Ave} := \frac{1}{\text{Area of } R} \iint_R f(x,y) dA = \frac{1}{\iint_R 1 dA} \iint_R f(x,y) dA.$$

$(*)$ One can imagine solid D as a block of ice in a cylindrical cup & melting, resulting flat-topped cylindrical cup of water.