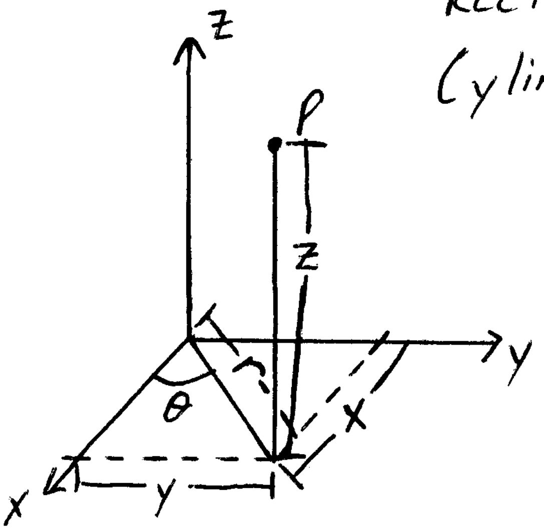
Math 21D
Vogler

Cylindrical Coordinates

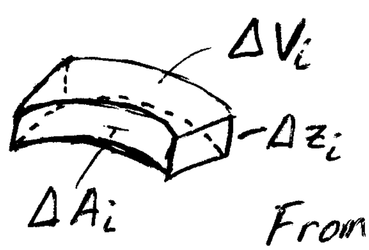
Rectangular Coordinates: $P=(x,y,z)$
Cylindrical Coordinates: $P=(r,\theta,z)$



Identities (From Polar Coords)

$x = r\cos\theta$
$y = r\sin\theta$
$r^2 = x^2+y^2$
$\tan\theta = y/x$

In Cylindrical Coordinates, our 'lego' piece looks like



To find the 'exchange rate' for dV, we note $\Delta V_i = (\text{base area})\cdot(\text{height}) = \Delta A_i \Delta z_i$.

From Polar Coordinates, $\Delta A_i = r\,\Delta r_i\,\Delta\theta_i$

$\Rightarrow \Delta V_i = r\,\Delta r_i\,\Delta\theta_i\,\Delta z_i = r\,\Delta z_i\,\Delta r_i\,\Delta z_i$

As $\Delta V_i \to 0$ (which is equivalent to $n\to\infty$), we conclude

$$\boxed{dV = r\,dz\,dr\,d\theta}$$

'Exchange Rate'

Notes: 1) $dV = r\,dz\,dr\,d\theta = r\,dr\,dz\,d\theta = r\,dz\,d\theta\,dr = r\,dr\,d\theta\,dz$
$= r\,d\theta\,dz\,dr = r\,d\theta\,dr\,dz$ are called <u>cylindrical</u> <u>coords</u>.

2) $dV = r\,dz\,dr\,d\theta$ is the most commonly used form, since it tends to be easiest to construct.

3) Cylindrical Coordinates are just Polar Coordinates with the third dimension given by the rectangular coordinate 'z' (i.e. measure of height)