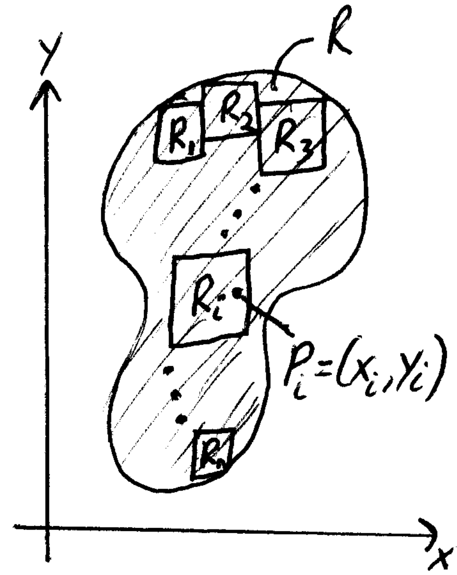
Math 21D
Vogler

Double Integrals



Defn

- Let function $f(P)$ be defined on region R.
- Let $R_1, R_2, \dots, R_n$ be subdivision of R with corresponding area $\Delta A_1, \dots, \Delta A_n$.
- Let $P_i = (x_i, y_i)$ be arbitrary point in R_i for all i.

Then the <u>definite</u> <u>integral</u> is

$$\int_R f(P)\,dA = \lim_{n\to\infty} \sum_{i=1}^{n} f(P_i)\,\Delta A_i$$

Notes: 1) Define the <u>mesh</u> as $\Delta A := \max_{1\le i\le n} \{\Delta A_i\}$

2) $n\to\infty$ is equivalent to $\Delta A \to 0$

3) The right hand side can be considered an infinite sum of infinitesmal multiplication.

4) dA can be thought of as an infinitly small 2D 'lego' piece.

5) $dA = dy\,dx$ (<u>vertical</u> <u>cross-sections</u>) & $dA = dx\,dy$ (<u>horizontal</u> <u>cross-sections</u>) are called <u>rectangular</u> <u>coordinates</u>.

Geometric Uses:

1) $\int_R 1\,dA$ = area of R

2) If $f(P)$ represents height of solid @ point P
$\Rightarrow \int_R f(P)\,dA$ = Volume of Solid (above region R)