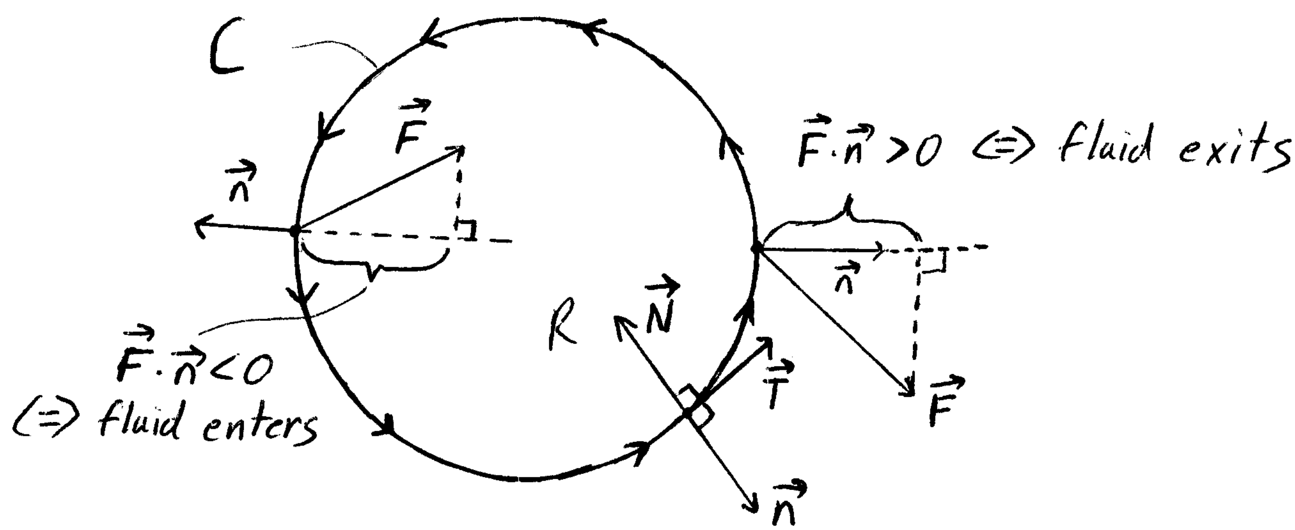
Vogler

Math 21D

Flux



Defn Let $\vec{F}(x,y) = M(x,y)\vec{i} + N(x,y)\vec{j}$ be a vector field defined on a smooth closed curve C in the plane. Let $\vec{n}$ be outward-pointing unit normal vector on C. The *flux* of $\vec{F}$ across C is

$$\text{Flux} = \int_C \vec{F}\cdot\vec{n}\, ds$$

Note: The unit normal vector $\vec{N}$ usually points in the opposite direction $\vec{n}$ (See above figure).

Method of Evaluation

Let C be curve defined by $\vec{r}(t) = g(t)\vec{i} + h(t)\vec{j}$, $a \le t \le b$, that traces C counterclockwise exactly once

$$\Rightarrow \text{Flux} = \int_C \vec{F}\cdot\vec{n}\, ds = \oint_C M\,dy - N\,dx = \int_a^b \left[M(\vec{r}(t))\frac{dg}{dt} - N(\vec{r}(t))\frac{dh}{dt}\right] dt$$

$$\text{or} \quad \text{Flux} = \int_a^b \left[M(\vec{r}(t))\frac{dx}{dt} - N(\vec{r}(t))\frac{dy}{dt}\right] dt$$