Math 21D
Vogler

Divergence, $\vec{k}$-component of Curl, & Green's Thm in the Plane

<u>Defn</u> Consider the vector field $\vec{F}(x,y) = M(x,y)\vec{i} + N(x,y)\vec{j}$. The <u>divergence of $\vec{F}$</u> at (x,y) is the scalar function

$$\text{div}\,\vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = M_x + N_y$$

Note: Divergence can be shown to be a measure of expansion ($\text{div}\,\vec{F} > 0$) or compression ($\text{div}\,\vec{F} < 0$) of a fluid or gas at a point (x,y)

<u>Defn</u> Consider the vector field $\vec{F}(x,y) = M(x,y)\vec{i} + N(x,y)\vec{j}$. The <u>$\vec{k}$-component of curl of $\vec{F}$</u> at (x,y) is the scalar function

$$(\text{curl}\,\vec{F})\cdot\vec{k} = \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = N_x - M_y$$

Note: $\vec{k}$-component of curl can be shown to be a measure of ~~co~~ counter-clockwise ($\text{curl}\,\vec{F}\cdot\vec{k} > 0$) fluid circulation or clockwise ($\text{curl}\,\vec{F}\cdot\vec{k} < 0$) fluid circulation at a point (x,y).

Green's Theorem 1 (Flux-Divergence or Normal Form)

Let $\vec{F} = M\vec{i} + N\vec{j}$ be a vector field defined on a region R enclosed by a simple closed curve C.

$$\Rightarrow \underbrace{\oint_C \vec{F}\cdot\vec{n}\,ds}_{\text{Outward Flux}} = \underbrace{\iint_R (M_x + N_y)\,dA}_{\text{Divergence Integral}}$$

Green's Theorem 2 (Circulation-Curl or Tangential Form)

Let $\vec{F} = M\vec{i} + N\vec{j}$ be a vector field defined on a region R enclosed by a simple closed curve C.

$$\Rightarrow \underbrace{\oint_C \vec{F}\cdot\vec{T}\,ds}_{\substack{\text{Counterclockwise}\\ \text{Circulation}}} = \underbrace{\iint_R N_x - M_y\,dA}_{\substack{\text{Curl }\vec{F}\cdot\vec{k}\\ \text{Integral}}}$$