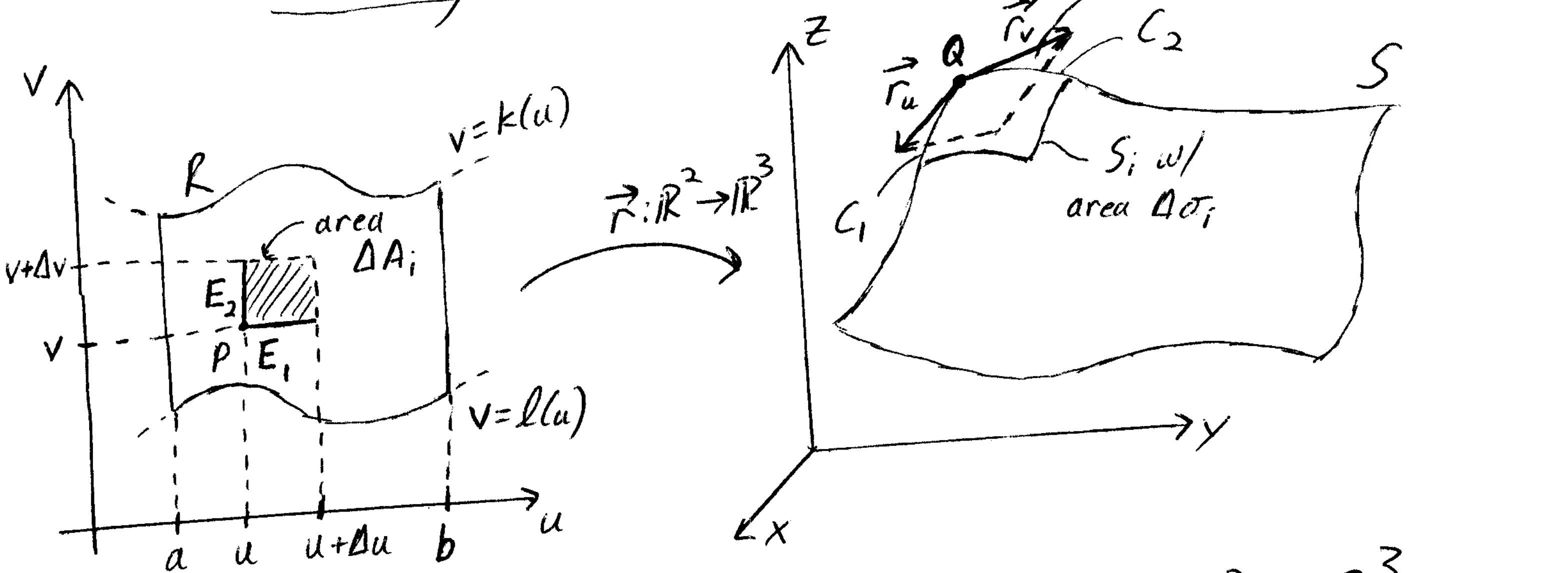
Math 21D
Vogler

Finding Area of Parametrized Surface



Let S be surface parametrized by $\vec r: R^2 \to \mathbb{R}^3$

$\vec r(u,v) = f(u,v)\vec i + g(u,v)\vec j + h(u,v)\vec k$ for (u,v)

in R: $a \le u \le b$, $\ell(u) \le v \le k(u)$.

Assume $\vec r(P) = Q$; let path C_1/C_2 be image of edge E_1/E_2 under mapping $\vec r$.

(Recall: $y = f(x) \Rightarrow \Delta f \approx f'(x)\Delta x$)

$\Rightarrow$ path $C_1 \approx \Delta\vec r$ (along E_1) $\approx \vec r_u(Q)\Delta u$

& path $C_2 \approx \Delta\vec r$ (along E_2) $\approx \vec r_v(Q)\cdot\Delta v$

Recall: Area of ▱ formed by vectors $\vec r_u \cdot \Delta u$ & $\vec r_v \Delta v$ is (magnitude of cross product)

$$|\vec r_u \cdot \Delta u \times \vec r_v \Delta v| = |\vec r_u \times \vec r_v| \underbrace{\Delta v \cdot \Delta u}_{\Delta A_i}$$

$$\Rightarrow \text{area } \Delta\sigma_i \approx \underbrace{|\vec r_u \times \vec r_v|}_{\text{'Exchange Rate'}} \Delta A_i$$ so that...

Defn If surface S is given by $\vec{r}(u,v): \mathbb{R}^2 \to \mathbb{R}^3$

$\vec{r}(u,v) = f(u,v)\vec{i} + g(u,v)\vec{j} + h(u,v)\vec{k}$ for (u,v) in R,

where $R: a \le u \le b,\ \ell(u) \le v \le k(u)$

$\Rightarrow$ Surface Area $S = \iint_R |\vec{r}_u \times \vec{r}_v| \, dA$

For Implicit Surfaces: (See book for derivations pg. 966 - 968)

- Let $F(x,y,z) = c$ be implicit surface above bounded plane region R

  $\Rightarrow$ Surface Area $= \iint_R \frac{|\vec{\nabla} F|}{|\vec{\nabla} F \cdot p|} \, dA$,

  where $\vec{p} = \vec{i}, \vec{j}$, or $\vec{k}$ is normal to R & $\vec{\nabla} F \cdot p \neq 0$

- Let $z = f(x,y)$ be over region R in xy-plane

  $\Rightarrow$ Surface Area $= \iint_R \sqrt{f_x^2 + f_y^2 + 1} \, dA$