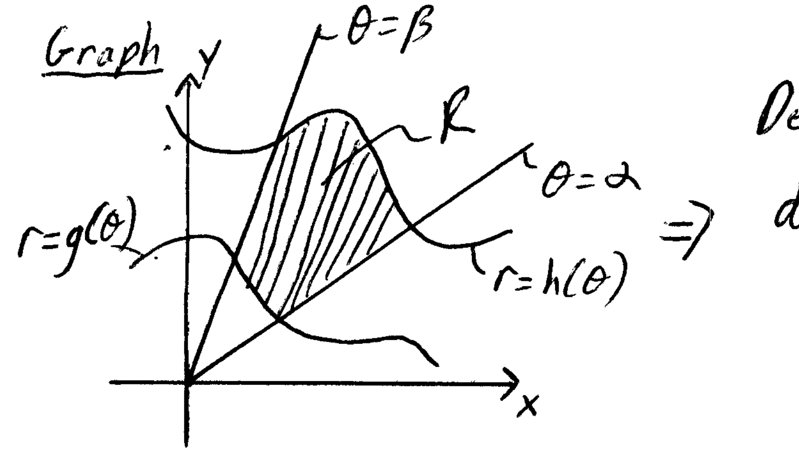
Math 21D
Vogler

# $\iint_R 1\, dA$ in Polar Coordinates

Consider the function $z = f(P) = f(r,\theta)$ (i.e. height function) over region $R$ bounded by graphs $\theta=\alpha$, $\theta=\beta$, $r=g(\theta)$ & $r=h(\theta)$

Graph

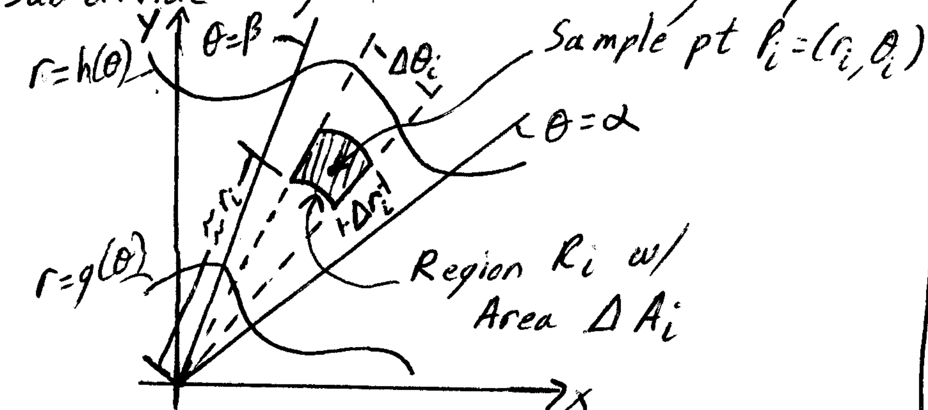



$\Rightarrow$ Described Region $R$:

$d\theta$: $\alpha \le \theta \le \beta$

$g(\theta) \le r \le h(\theta)$

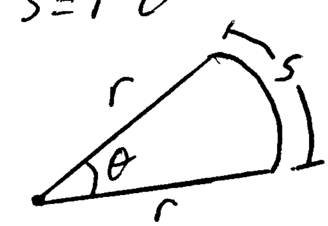
Subdivide region $R$ in following way:




**Recall:**

Circular Arc Length

$s = r\theta$




Hence, area of $R_i$ (approximately a rectangle)

$$\Delta A_i \simeq s\cdot \Delta r_i = r_i\,\Delta\theta_i\,\Delta r_i = r_i\,\Delta r_i\,\Delta\theta_i$$

$$\Rightarrow \iint_R f(P)\,dA := \lim_{n\to\infty}\sum_{i=1}^{n} f(P_i)\Delta A_i = \lim_{n\to\infty}\sum_{i=1}^{n} f(r_i,\theta_i)\,r_i\,\Delta r_i\,\Delta\theta_i$$

$$=: \int_\alpha^\beta \int_{g(\theta)}^{h(\theta)} f(r,\theta)\, r\,dr\,d\theta$$

We conclude: In Polar Coordinates $\boxed{\iint_R f(P)\,dA = \int_\alpha^\beta\int_{g(\theta)}^{h(\theta)} f(r,\theta)\,r\,dr\,d\theta}$

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Notes: 1) $dA = r\,dr\,d\theta$

2) $dx\,dy = r\,dr\,d\theta$ & the 'r' can be thought of the 'exchange rate' of converting 'lego pieces' to ones.