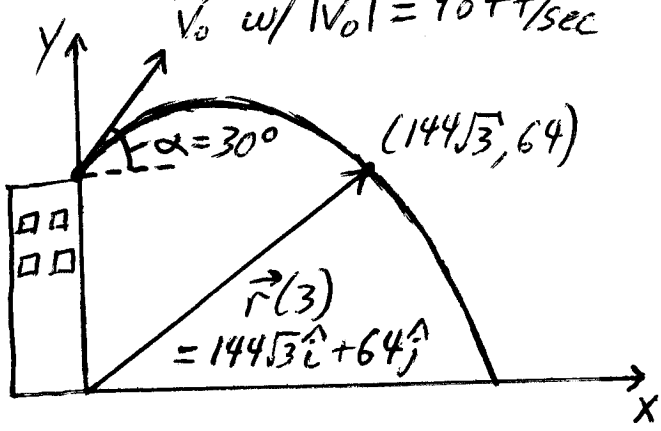
Math 210
Vogler

Projectile Motion Ex

Ex A golf ball is hit @ a 30° angle to the horizontal with initial velocity 98 ft/sec off the top of a building 64 ft high.



Equations

$x(t) = |\vec{v}_0|\cos\alpha\, t = (96\cos 30^\circ)t$
$= 48\sqrt{3}\,t$

$y(t) = b + |\vec{v}_0|\sin\alpha\, t - \frac{1}{2}gt^2$
$= 64 + (96\sin 30^\circ)t - 16t^2$
$= 64 + 48t - 16t^2$

Hence, $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$ w/ $\boxed{x(t) = 48\sqrt{3}\,t \;\&\; y(t) = 64 + 48t - 16t^2}$

1) What's position of ball @ $t = 3$ secs?

$\vec{r}(3) = x(3)\hat{i} + y(3)\hat{j} = 48\sqrt{3}(3)\hat{i} + (64 + 48(3) - 16(3)^2)\hat{j} = \boxed{144\sqrt{3}\,\hat{i} + 64\hat{j}}$

2) How high does the ball go?

The highest point is equivalent to $y'(t) = 0$ for certain t.

$\Rightarrow$ Want to find t such that $y'(t) = 48 - 32t = 0$

$\Rightarrow$ $t = 3/2$ sec $\Rightarrow$ Max. height $= y(3/2) = 64 + 48(\frac{3}{2}) - 16(\frac{3}{2})^2$
$= \boxed{100\text{ ft}}$

3) How long is it in the air?

Time t it strikes ground is equivalent to $y(t) = 0$

$\Rightarrow y(t) = 64 + 48t - 16t^2 = 0 \Rightarrow -16(t^2 - 3t - 4) = 0 \Rightarrow (t-4)(t+1) = 0$

$\Rightarrow \boxed{t = 4\text{ sec}}$ or ~~$t = -1$ sec~~

4) How far from base of building land? $x(4) = 48\sqrt{3}(4) = \boxed{192\sqrt{3}\text{ ft}}$

5) What's ball's direction of motion & speed as it strikes the ground?

$\vec{r}(t) = 48\sqrt{3}\,t\,\hat{i} + (64 + 48t - 16t^2)\hat{j} \xrightarrow{D} \vec{v}(t) = \vec{r}\,'(t) = 48\sqrt{3}\,\hat{i} + (48 - 32t)\hat{j}$

- Direction of motion @ $t = 4 = \vec{v}(4) = 48\sqrt{3}\,\hat{i} - 80\hat{j}$
- Speed @ $t = 4 = |\vec{v}(4)| = \sqrt{(83)^2 + (-80)^2} \approx 115$ ft/sec