MAT 17A - DISCUSSION #1
September 29, 2015

Geometric and Allometric Scaling in Biology

PROBLEM 1. A DANGER OF BEING SMALL: GETTING WET.

"A man coming out of a bath carries with him a film of water about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as everyone knows, a fly once wetted by water or any other liquid is in a very serious position indeed."

- J.B.S. Haldane in "On Being the Right Size" in The Harper's Monthly, March 1926.

How do you think Haldane came up with these conclusions? Did he go out and weigh humans, mice, and flies before and after dipping them in water? Probably not. In fact, these statements are probably not that precise. The main point of Haldane's statement is that as you get smaller the more dangerous getting wet becomes. Let's build an idealized mathematical model to see why this is.

(i) Let R denote the ratio between the mass of the water film clinging to an animal's body after getting wet **(**M_W) and the dry animal's body mass (M_B). Find an expression (i.e., a mathematical model) that approximates R as a function of M_B. Sketch a graph of R as a function of M_B.

* Why is R as a function of M_B something we'd like to know?

* Construct your model by making simplifying assumptions, including:

- assume that all animals are approximately the same shape, and model the body of an animal as a simple geometrical shape (i.e., a sphere, a cylinder, etc.).
- assume that the mass of the water film on a wet animal is proportional to the animal's body surface area, and use Haldane's estimate of the thickness of the water film ($l = \frac{1}{50} in = 0.05\ cm$).
- assume that the density of an animal is the same as the density of water.

(ii) Compute R for a human (60 kg), a cat (5 kg), a rat (0.25 kg), mouse (0.02 kg), a shrew (0.004 kg), a bee (0.0001 kg), a housefly (0.00002 kg), and a mosquito (0.0000025 kg). Plot the data points corresponding to these animals on your graph R vs M_B from (i).

(iii) What can you conclude from your mathematical analysis? Do your calculations agree with Haldane's assertions? Do you see any issues with your/Haldane's conclusions that getting wet poses serious dangers to small animals?

(iv) Consider the assumptions that you made in coming up with your mathematical model. Are they reasonable assumptions? How could you make your model more precise? Do you think your basic conclusions would change significantly if you constructed a more precise (but probably more complicated) model?

PROBLEM 2: ENERGY COMSUMPTION AND SIZE OF ANIMALS

Basal metabolic rate (BMR) is the minimal rate of energy expenditure per unit time by endothermic animals at rest. That is, the BMR is the amount of energy that an animal needs to keep the body functioning at rest through processes such as breathing, blood circulation, controlling body temperature, cell growth, brain and nerve function, and contraction of muscles. A human's basal metabolic rate typically accounts for about 60 to 75% of the calories they burn every day.

	approximate body weight, W (lbs)	**basal metabolic rate, BMR (kcal/day)**
elephant	**10000**	**18000**
horse	**1000**	**3200**
human	**200**	**960**
cat	**8**	**86**
rat	**2**	**30**
mouse	**0.5**	**10**

The table above lists the body weight (W) and basal metabolic rate (BMR) of several different mammals. Plot this data using R and try to determine the functional relationship (scaling law) between BMR and W.

To do this, type the follow R commands in the console of R Studio (after the > prompt):

```
w = c(10000, 1000, 200, 8, 2, 0.5)

bmr = c(18000, 3200, 960, 86, 30, 10)

plot( w, bmr, xlab="mass (lbs)", ylab="BMR (kcal/day)" )
```

... or for a slightly fancier plot, replace the last command with

```
plot(mass, bmr, xlab="weight (lbs)", ylab="BMR(kcal/day)", cex=1.25, pch=21, bg="red", lwd=1)
```

(Try to figure out what each command does.)

Questions to think about:

Examine the graph of BMR and weight that you have just plot in R. Can you identify the relationship between an animal's BMR and weight? Can you even distinguish all of the data points? What is preventing you from doing this? Can you think of a way to remedy the situation?