MAT 17A - DISCUSSION #2
October 6, 2015

PROBLEM 1. log rules!

Suppose a, b, c, and d are arbitrary positive constants. Using the identities

$$b^a\, b^c = b^{a+c}\,, \quad \text{[distributive law for exponents]}$$
$$\log_{10}(10^d) = d\,, \quad \text{[definition of log, i.e., inverse of exponential function]}$$

show that, for any positive constants x and y,

$$\log_{10}(x\,y) = \log_{10}(x) + \log_{10}(y).$$

(Hint: write x and y as $x = 10^s$ and $y = 10^t$, where you must determine what s and t are).

PROBLEM 2. Weight and wingspan of birds - "Why did dodos go extinct?" and "What should the wingspan of a flying human be?"

Ornithologists have measured and cataloged the wingspans and weights of many different species of birds. The table below shows the wingspan L for a bird of weight W.

(a) Use *R Studio* to make a scatter plot of the data. Make a semi-log plot, and a log-log plot. Are there any outliers in the data? Is there any justification for removing them from the set?

(b) Find an exponential function model and a power function model for the data (with outliers removed).

(c) Graph the models from b). Which fit appears better?

(d) The dodo is a bird that has been extinct since the late 17th century.* It weighed about 45 pounds and had a wingspan of about 20 inches. Why couldn't the dodo fly? For what reason(s) might the dodo have gone extinct? What data backs up your hypothesis?

(e) Based on your model, what wingspan would we need to be flying-humans?

[see appropriate R commands on last page.]

BIRD	**average body weight, W (lb)**	**average wingspan, L (in)**
Turkey vulture	**4.40**	**69**
Bald eagle	**6.82**	**84**
Great horned owl	**3.08**	**44**
Cooper's hawk	**1.03**	**28**
Sandhill crane	**9.02**	**79**
Atlantic penguin	**0.95**	**24**
King penguin	**29.0**	**28**
California condor	**17.8**	**109**
Common loon	**7.04**	**48**
Yellow warbler	**0.022**	**8**
Emu	**138**	**69**
Common grackle	**0.20**	**16**
Wood stork	**5.06**	**63**
Mallard	**2.42**	**35**
*Dodo**	*45*	*20*

PROBLEM 3. Fermi problem* of the week

How many golf balls can fit in a school bus?

(This question is rumored to have been asked in job interviews at Google.)

PROBLEM 4. Sequences and limits

(a) List the first five to ten terms of the sequence and plot a graph of the sequence by hand

$$(i) \quad a_n = \frac{2\,n}{n^2 + 1}$$

$$(ii) \quad b_k = 2 + \frac{(-1)^k}{k}$$

(b) Consider the function

$$h(x) = \frac{2\,x}{x - 1}$$

(i) Use *data tables* to find out what is happens to $h(x)$ as x get larger and larger in the positive direction, and as x get larger and larger in the negative direction? What happens as x gets closer and closer to 1 (from above and below)?

(ii) Based on your answers in (i), what are $\lim_{x\to\infty} h(x)$, $\lim_{x\to-\infty} h(x)$, and $\lim_{x\to 1} h(x)$?

(iii) Based on your answer in (ii), sketch $h(x)$.

PROBLEM 5. Discon tinuities

Sketch the following functions and discuss the different types of discontinuities that they exhibit.

$$(i) \quad h(x) = \frac{2x}{x - 1}$$

$$(ii) \quad f(t) = \frac{t^2 - 1}{t + 1}$$

$$(iii) \quad w(y) = \begin{cases} y - 1, & y \geq 0 \\ -2y, & y < 0 \end{cases}$$

$$(iii) \quad p(x) = \cos\left(\frac{1}{x}\right)$$

R CODE FOR PROBLEM 2

Type the uncommented lines of R-code (i.e., the lines without "#" at the beginning) in the console window.

```
# A. INPUT DATA AND PLOT

# data is listed in this order:
#  (Turkey vulture, Bald eagle, Great horned owl, Cooper's hawk, Sandhill crane, Atlantic puffin, King penguin,
#  California condor, Common loon, Yellow warbler, Emu, Common grackle, Wood stork, Mallard, Dodo*)

# corresponding Weights of birds
w=c(4.4,6.82,3.08,1.03,9.02,0.95,29,17.8,7.04,0.022,138,0.2,5.06,2.42,45)

# corresponding wingspan of birds
s=c(69,84,44,28,79,24,28,109,48,8,69,16,63,35,20)

#plot data
plot(w,s,xlab="weight (lbs)",ylab="wingspan (in)")

# B. TAKE LOG OF DATA (BOTH w and s) AND RE-PLOT

# take log_10 of weight and wingspan data
wlog=log10(w)
slog=log10(s)

#plot log-log data on a new graph
plot(wlog,slog,xlab="log(weight)",ylab="log(wingspan)")

# What linear function y = a x + b would be a good fit to the log(wingspan) vs log(weight) data
# except for outliers (penguin, emu and dodo), where x = log(weight) and y = log(wingspan)?

# C. CREATE A LINEAR FUNCTION WITH SLOPE "a" and VERTICAL INTERCEPT "b" AND PLOT THE
#    CORRESPONDING LINE ON A FIGURE THAT ALREADY EXISTS.

# change parameters "a" and "b" so that the line will fit your data
a = 2
b = -1

# create a sequence of numbers "x" from "xmin" to "xmax" with steps of "xstep".
xmin = -2
xmax = 2.5
xstep = 0.1
x=c(from = xmin, to = xmax, by = xstep)

# create a corresponding sequence of numbers "y=ax+b" for each value in the sequence "x"
y=a*x+b

# plot the curve going through points (x,y) on top of the graph that you already have plotted
lines(x,y,"l")
```