MAT 17A - DISCUSSION #3
October 13, 2015

Problem 1. Measuring Diaphragm Change as a Function of Pressure [FAKE DATA VERSION]

When you take a breath, a muscle called the diaphragm reduces the pressure around your lungs, which expand to fill with air. The table below shows the volume of a lung (V) as a function of the reduction in pressure from the diaphragm (P). Pulmonologists define the compliance of the lungs as the ratio of the change in volume (ΔV) over the change in pressure reduction, i.e., $\frac{\Delta V}{\Delta P}$ = compliance.

Use the table below and *R/RStudio* to answer the following questions.

pressure reduction (cm of water)	volume (liters)
0	0.10
5	0.20
10	0.30
15	0.40
20	0.50
25	0.60
30	0.70

(a) What are the units of compliance?

(b) Use *R/RStudio* to plot lung volume (V) versus reduction in pressure from the diaphragm (P).

You can store P with the command:

```
> P = c(0,5,10,15,20,25,30)
```

Store V similarly, then plot using the command:

```
> plot(P,V,"b",xlab="pressure reduction (cm of water)",ylab="volume (liters)")
```

(c) Find the compliance for each pressure reduction value (P). Plot compliance versus P using *R/RStudio*.

pressure reduction (cm of water)	change in pressure reduction (ΔP) (cm of water)	change in volume (ΔV) (liters)	compliannce ( )
2.5			
7.5			
12.5			
17.5			
22.5			
27.5			

(d) When type of relationship is V versus P? How does the compliance depend on P? Do your answers to these two questions make sense, i.e., is there an appropriate correspondence between them?

Problem 2. Measuring Diaphragm Change as a Function of Pressure

When you take a breath, a muscle called the diaphragm reduces the pressure around your lungs, which expand to fill with air. The table below shows the volume of a lung (V) as a function of the reduction in pressure from the diaphragm (P). Pulmonologists define the compliance of the lungs as the instantaneous rate of change of the volume of a lung with respect to changes in the reduction in, i.e., compliance is the derivative of V with respect to P : compliance = $\frac{dV}{dP}(P)$. It is important to note that lung compliance is typically *not* constant but depends on the pressure reduction P.

Use the table below and *R/RStudio* to answer the following questions.

pressure reduction (cm of water)	volume (liters)
0	0.20
5	0.29
10	0.49
15	0.70
20	0.86
25	0.95
30	1.00

(a) What are the units of compliance?

(b) Use *R/RStudio* to plot lung volume (V) versus reduction in pressure from the diaphragm (P).

You can store P with the command:

```
> P = c(0,5,10,15,20,25,30)
```

Store V similarly, then plot using the command:

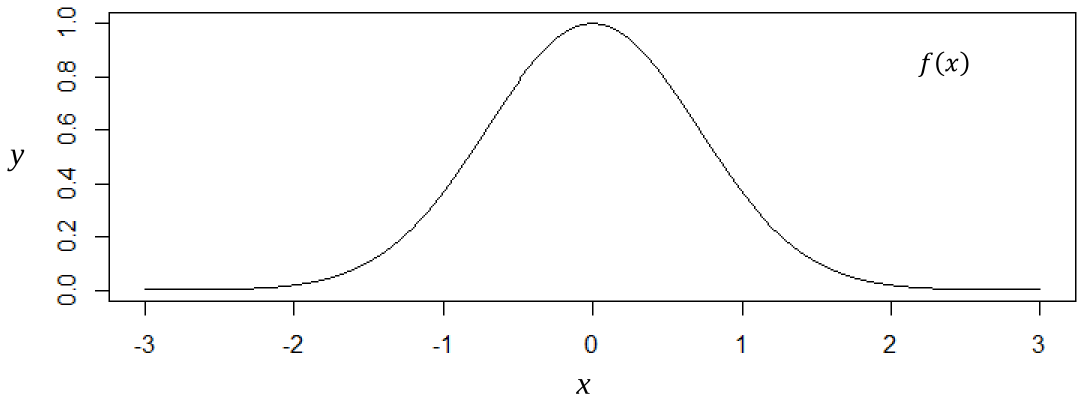
```
> plot(P,V,"b",xlab="pressure reduction (cm of water)",ylab="volume (liters)")
```

(c) Estimate the compliance for each pressure reduction value (P). Plot compliance versus P using *R/RStudio*.

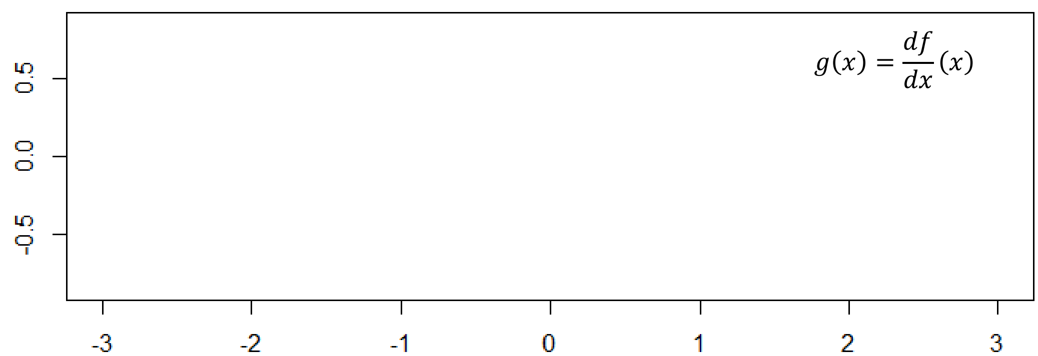
pressure reduction (cm of water)	change in pressure reduction (ΔP) (cm of water)	change in volume (ΔV) (liters)	est. compliannce ( )
2.5			
7.5			
12.5			
17.5			
22.5			
27.5			

(d) When is compliance largest? What is occurring physiologically at this moment? Why does compliance decrease near the value of P corresponding to V =1.00?

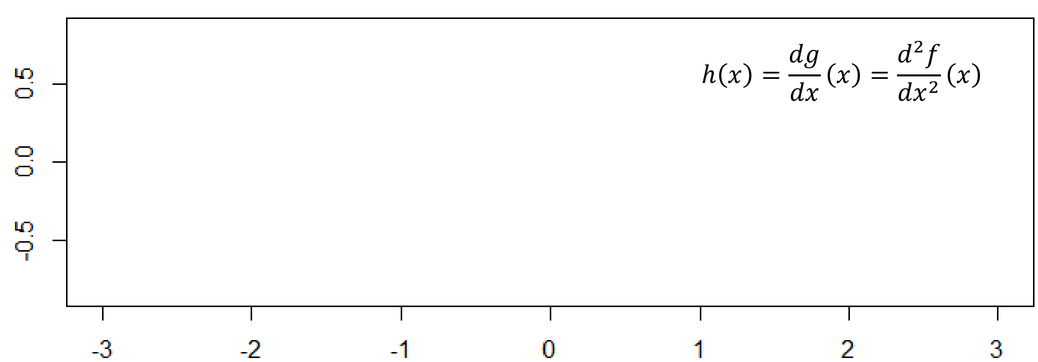
PROBLEM 3. Consider the function $y = f(x)$ graphed below.



(a) Sketch the derivative of $f(x)$. Be sure to label the axes. What does the derivative of $f(x)$ tell you about the function $f(x)$ itself?



(b) Sketch the derivative of the derivative of $f(x)$, i.e., the second derivative of $f(x)$. Again, be sure to label the axes. What does the second derivative of $f(x)$ tell you about the function $f(x)$ itself?



PROBLEM 4. Spread of a Virus

The spread of a virus is modeled by

$$I(t) = -t^2 + 6t - 4,$$

where $I(t)$ is the number of people (in hundreds) infected with the virus and t is the number of weeks since the first case was observed.

(a) Graph $I(t)$.

(b) What is a reasonable domain for $I(t)$ (including units), i.e., what values of t makes sense as input for $I(t)$?

(c) When does the number of infected individuals reach a maximum? What is the maximum number of cases?

(d) Use the limit definition of the derivative to find the function which describes the rate of change of the number of infected individuals? Then find $\frac{dI}{dt}$ at $t = 2$ and identify the units of this value. What do these units mean?

(e) What is the rate of change in the number of cases at the maximum? How does this value compare with the phenomenon occurring at this time?

(f) Give the sign (+ or −) of the rate of change up to the maximum and after the maximum.

PROBLEM 5. Fermi problem of the week

How many hairs are there on your head?