MAT 17A - DISCUSSION #4

October 20, 2015

Problem 1. Plotting a Function in R/Rstudio

Suppose you want to plot the function

$$f(x) = x^2 + 3.$$

Of course, you could choose various values of x, store them in a vector using

```
x = c(-4,-3,-2,-1,0,1,2,3,4)
```

To find the corresponding function values, you could compute the corresponding values for each x value and store them in a vector using

```
y = c(19,12,7,4,3,4,7,12,19)
```

Now, you plot using:

```
plot(x,y, xlab= ‘x values’, ylab = ‘function values’)
```

However, you could also use R's built-in "vector" manipulation tools to compute and plot these vectors as follows

```
x = seq(from = -4, to = 4, by = 1) #a list of integers from -4 to 4
y = x^2 + 3       #a list of the corresponding function f(x) values
plot(x,y, xlab = ‘x values’, ylab = ‘function values’)
```

Q: What happens when you change the number inputs for ‘from’, ‘to’ and ‘by’?
Q: What happens when you change the form of the function for y?

Problem 2. Ant biodiversity

Noting that the number of ant species declines at both low (close to sea level) and high (at the tops of mountains) levels, ecologists fitted a parabola to data. The fit they obtained is given by

$$S = S(y) = -10.3 + 24.9y - 7.7y^2 \ ,$$

where y is elevation measured in kilometers and S is the number of species.

(a) Plot $S(y)$ for $-0.085 \leq y \leq 6.168$ using R/Rstudio. Use the graph to estimate the range of number of ant species across North America (see note below).

(b) For what elevations does this model make sense? Replot the function on this elevation interval.

(c) Find $\frac{dS}{dy}(y)$, using (i) the limit definition of the derivative, and (ii) differentiation rules.

(d) Discuss and interpret the units of $\frac{dS}{dy}(y)$.

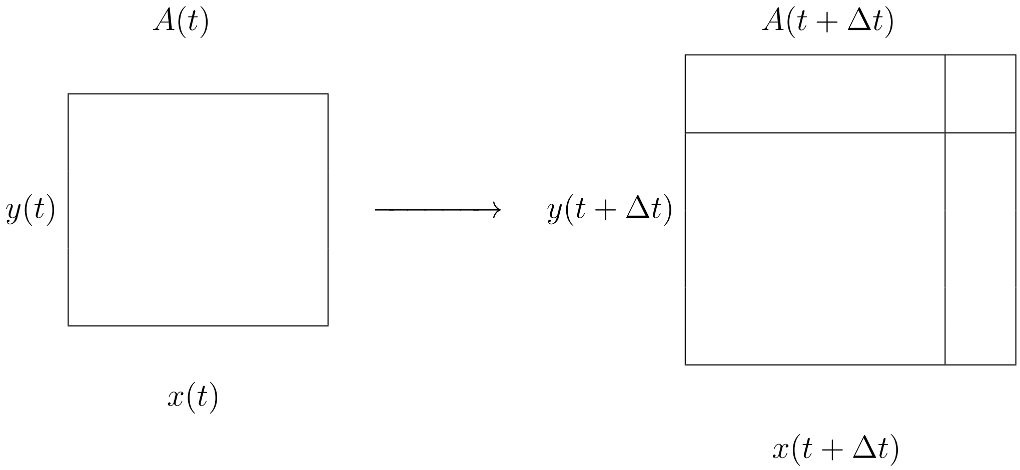
(e) Plot $\frac{dS}{dy}(y)$ for interval of y that you found in (b) using R/Rstudio.

(f) At which elevation is the number of ant species changing most rapidly and what is the associated rate of change?

Note: The lowest point in North America is Badwater Basin in Death Valley, which is 85 meters below sea level. The highest point is Mount McKinley in Alaska at 6,168 meters above sea level.

Problem 3. The Derivative of the Product of two Functions

Here, we will derive the rule for taking the derivative of the product of two functions, i.e., $\frac{d}{dt}(x(t)y(t))$.



Consider a rectangle whose lengths are changing in time, as in the diagram above. At time t, the area of the rectangle is $A(t)$ and the lengths of the sides of the rectangle are $x = x(t)$ and $y = y(t)$. At time $t + \Delta t$, the area of the rectangle is $A(t+\Delta t)$ and the lengths of the sides of the rectangle are $x(t+\Delta t) = x+\Delta x$ and $y(t+\Delta t) = y+\Delta y$. (Note that the terms $x(t+\Delta t)$ and $y(t+\Delta t)$ indicate the full length of each side of rectangle on the right-hand side of the diagram.)

(a) Fill in the diagram above by doing the following:

(1) On the rightward arrow, label the top Δt.

(2) Label each *area* in the right rectangle using the terms x, y, Δx, and Δy.

(b) Compute the change in area, ΔA, based on the diagram. Your expression should be only in terms of $x, y, \Delta x$ and Δy:

$\Delta A =$

(c) Find the ratio of rate of change of area with respect to time, $\frac{\Delta A}{\Delta t}$:

Make every term with a delta (Δ) in the numerator should have a delta in the denominator.

$$\frac{\Delta A}{\Delta t} =$$

(d) Using your expression for $\frac{\Delta A}{\Delta t}$, compute the average rate of change of area with respect to time as $\Delta t \longrightarrow 0$

$$\frac{dA}{dt} = \lim_{\Delta t \longrightarrow 0} \frac{\Delta A}{\Delta t} =$$

(e) Based on your work above, what is

$$\frac{d}{dt}(x(t)y(t)) =$$

Problem 4. Think"Rate of Change" - Limit Definition of the Derivative
Remembering that the limit definition of the derivative of a function, f, is

$$f'(x) = \lim_{\triangle x \to 0} \frac{f(x + \triangle x) - f(x)}{\triangle x},$$

how can you interpret this definition? What is the numerator of the fraction inside the limit? The denominator? Thus, what is the ratio within the limit?

Draw a picture of a non-linear function and using several choices of $\triangle x$, indicate pictorially what the ratio within the limit represents.

Problem 5. A very special function ...

Try to draw a function whose derivative is equal to itself, i.e., find $y = f(x)$ such that $\frac{df}{dx}(x) = f(x)$ or in different notation $\frac{dy}{dx} = y$.

Problem 6. Fermi problem of the week

How many cells are there in your body?