MAT 17A - DISCUSSION #5

October 27, 2015

Problem 1.

Suppose $f(6) = 5$, $f'(6) = 4$, $g(6) = 6$, and $g'(6) = 4$. Find $(f(x)g(x))'$ at $x = 6$.

Problem 2. Survival Rates

A zebra in Etosha National Park, Namibia, can die either because it is killed by a predator or because it succumbs to disease (primarily anthrax). Starting at the beginning of each year, suppose $f(t)$ and $g(t)$ respectively represent the probabilities of surviving predation and disease by the end of week t.

Prof. Wayne Getz's research group (UC Berkeley) determined that at the height of a typical anthrax season, which occurs around the end of the eleventh week of a typical year (i.e., $t = 11$), $f(11) = 0.965$ and $g(11) = 0.981$. They also determined that the probability of surviving predation and disease at the end of the eleventh week is decreasing by 0.004 and 0.005 per week, respectively.

Assume that the events of dying from predation or disease are independent, so that the probability of surviving both predation and disease is $f(t)g(t)$.
(a) Find the probability of surviving to the end of week 11.

(b) Find the rate at which this probability is changing at the end of week 11.

(c) Use your answers from parts (a) and (b) to estimate the probability of surviving to the end of week 12.

Problem 3. Olympic Weightlifting

We can model the amount an Olympic weightlifter can lift as

$$L = 20.15M^{2/3} \text{ kilograms}$$

where M is the body mass in kilograms of the weightlifter. Find and interpret $\frac{dL}{dM}$.

You can plot a graph of L with the R commands below:

```
> M = seq(68,159,by=.5)
> L = 20.15*M^(2/3)
> plot(M,L,xlab='Mass of Weightlifter',ylab='Amount Able to Lift')
```

Now, plot a graph of $\frac{dL}{dM}$ by the commands above (and be sure to properly label your axes)!

(a) What is an interpretation of $\frac{dL}{dM}$ at $M = 90kg$?

(b) Convert the above relations of L and M and $\frac{dL}{dM}$ and M to represent the amount a weightlifter can lift in pounds given his body mass in pounds and replot.

(c) Plot $\frac{2}{3}\frac{L}{M}$ on the same plot as $\frac{dL}{dM}$. What do you notice about these plots? Can you explain what is happening?

Problem 4. Population Density

Shenzhen, which is a major city in southern Chinas Guangdong Province, is one of the fastest growing cities in the world. In 1996, area of Shenzhen was 600 km^2 and expanding at a rate of 40 $km^2/year$. The population of Shenzhen in 1996 was 5,000,000 people, and it was growing at a rate of 400,000 *people/year*.

(a) Determine the population density of Shenzhen in 1996.

(b) Use the appropriate derivative rule and the data above to determine how fast the population density of Shenzhen was growing in 1996?

(Include the appropriate units).

Problem 5. Protein Concentration in Cells

Consider a growing bacterial cell that is producing a protein involved in binary fission. Suppose that, at a particular point in time, a bacterial cell has a volume of 1000 μm^3 (microns cubed) and is growing at a rate of 5 $\mu m^3/min$. Furthermore, suppose that there are 1000000 molecule of the protein in the cell, and the protein is accumulating at a rate of 10000 molecules/min.

Find the concentration of the protein in the cell and the rate at which the concentration of protein is changing within the cell (include units). Is the concentration increasing or decreasing? Thoroughly justify your answer with the appropriate derivative rule.

Problem 6. Fermi

How many minutes do you spend on your phone each year (including talking, texting, etc)?

(Challenge:) How many minutes does the average American spend on their phone each year?