Math 17A
Vogler
Discussion Sheet 6

1.) Use any method to differentiate the following functions. You need not simplify answers.

a.) $y = \sqrt{x^2 - 3x - 4}$ b.) $f(x) = (x^3 + (x^2 + 1)^3)^4$ c.) $f(x) = \dfrac{(x+5)^3}{(2-x)^4}$

d.) $g(x) = \left(\dfrac{x+1}{x^2+1}\right)^{20}$ e.) $f(x) = \sqrt{1 + \sqrt{2 + \sqrt{3 + x}}}$

2.) Let $f(x) = x(x-5)^4$.

a.) Solve $f'(x) = 0$ for x.
b.) Solve $f''(x) = 0$ for x.

3.) Assume that a pomegranate is projected directly upward from the ground with an initial velocity of 112 ft./sec. It can be shown that the pomegranate's height above the ground at time t seconds is given by $s(t) = -16t^2 + 112t$ feet.

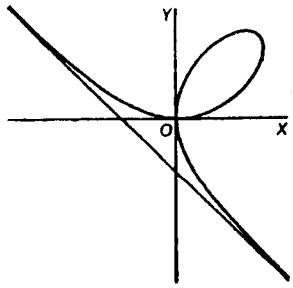
a.) What is the height of the pomegranate after $t = 1$ sec., $t = 2$ sec., and $t = 5$ sec.?
b.) What is the velocity of the pomegranate after $t = 1$ sec., $t = 2$ sec., and $t = 5$ sec.?
c.) At what time does the pomegranate reach its maximum height ? What is the pomegranate's maximum height ? In how many seconds does the pomegranate strike the ground ?
d.) How long is the pomegranate in the air ? What is the pomegranate's velocity as it strikes the ground ?
e.) What is the pomegranate's acceleration when $t = 1$ sec., $t = 2$ sec., $t = 5$ sec. ?

4.) Assume that y is a function of x. Compute $y' = \dfrac{dy}{dx}$ and $y'' = \dfrac{d^2y}{dx^2}$ (You need not simplify y''.) for each equation. Write all answers in terms of x and y only.

a.) $y = x^2 + x$
b.) $x = y^2 + y$
c.) $x^2 + y^3 = xy$

5.) Use implicit differentiation to find the largest y-value in the "loop" of the Folium of Descartes, which is given by the equation $x^3 + y^3 - 3xy = 0$ (See diagram below.).

Folium of Descartes



6.) Find the slope of the graph of $xy^2 + y = 2$ at $x = 0$ and at $x = 1$. Sketch the graph near $x = 0$ and $x = 1$.

7.) The volume V of a sphere is changing at the rate of π ft.^{3}/min. At what rate is the sphere's surface area S changing when

a.) S $= 4\pi$ ft.^{2} ? b.) S $= 36\pi$ ft.^{2} ?

(RECALL: For a sphere volume $V = (4/3)\pi r^3$ and surface area $S = 4\pi r^2$.)

8.) Car B is 30 miles directly east of car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph. At what rate is the distance between the cars changing after $t = \frac{1}{5}$ hr. ? $t = \frac{1}{3}$ hr. ?

9.) A conical tank (point down) has height 10 ft. and base radius 8 ft. Water begins flowing into the tank at the rate of π ft.^{3}/sec. At what rate is the depth h of the water changing

a.) when $h = 1$ ft. ? b.) when $h = 9$ ft. ?

(RECALL: For a cone volume $V = (1/3)\pi r^2 h$.)

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The following problem is for recreational purposes only.

10.) Find all points (x, y) which are equidistant from the three given points (0, 0), (4, 0), and (3, 2) .

Thanks to Dr. Kouba