Math 17A
Vogler
Discussion Sheet 7

1.) Solve for x.

a.) $\ln x = 3$ b.) $\ln x = \ln 3$ c.) $\ln(2x+1) - \ln(x+3) = 0$
d.) $\ln(x-1) + \ln(x-2) = 0$ e.) $\ln(x-2) + \ln(x+2) - \ln x = \ln 3$

2.) Let $f(x) = x^3 \ln x$. Solve $f'(x) = 0$ and $f''(x) = 0$ for x and set up a sign chart for each.

3.) Find $y' = \dfrac{dy}{dx}$ as simply as possible. Do not simplify your answers.

a.) $y = \ln(5x+7)$ b.) $y = x^{\ln 7}$ c.) $y = \ln(x \tan x)$
d.) $y = \ln\left(\dfrac{x^2}{x^2+3}\right)$ e.) $y = \ln(\ln(\ln(\sin x)))$

4.) Solve for x.

a.) $e^x = 2$ b.) $7 \cdot e^{2x+3} = e^{3-x}$ c.) $(e^x+1)(e^x-5) = 0$
d.) $e^{2x} - 4 \cdot e^x = 0$ e.) $e^{2x} - 5 \cdot e^x + 6 = 0$

5.) Let $f(x) = x^2 e^{-x}$. Solve $f'(x) = 0$ and $f''(x) = 0$ for x.

6.) Find $y' = \dfrac{dy}{dx}$. Do not simplify your answers.

a.) $y = 7 \cdot e^{5x-4}$ b.) $y = e^{x^2} \cdot \tan(3x)$ c.) $y = \dfrac{2^x + 4^x}{3^x + 5^x}$

7.) Assume that y is a function of x. Compute $y' = \dfrac{dy}{dx}$ and $y'' = \dfrac{d^2y}{dx^2}$ (You need not simplify y''.) for each equation.

a.) $y = e^{x^2}$ b.) $x = e^{y^2}$ c.) $y = \ln(3x-2)$ d.) $x = \ln(3y-2)$
e.) $y = \ln(3x-y)$ f.) $\sin x = y - \cos xy$ g.) $xy^2 = x + \tan y$

8.) Assume that $x^2 + (5-y)^3 = 2x + 125$.

a.) Find $y' = \dfrac{dy}{dx}$ at the point $(0,0)$ using implicit differentiation.

b.) Solve the original equation for y. Then find $y' = \dfrac{dy}{dx}$ at the point $(0,0)$ using ordinary differentiation.

9.) One hundred (100) years ago the gold mining town of Prospect, CA, had a population of 15,530 people. Today the population is 3750. Assuming exponential decay, what do you predict the population of Prospect will be 50 years from now ?

10.) Assume that a fossilized bone found today contains 35% of it original amount of carbon-14. If the half-life of carbon-14 is 5730 years, estimate the age of the fossil.

11.) Use logarithmic differentiation to differentiate each function.

a.) $y = x^{\ln x}$ b.) $y = x^{e^x}$ c.) $y = \dfrac{x^4(x-3)^5 \sin x}{e^x\sqrt{x^2+x}}$

12.) Let $f(x) = x^3 + x + 5$.

a.) Use a derivative sign chart to verify that f is one-to-one. Thus, $y = f(x)$ has an inverse function, $y = f^{-1}(x)$.

b.) Find the derivative of the inverse function for $x = 7$, i.e., find $D\{f^{-1}(7)\}$

13.) Consider the exponential function $N(t) = Ce^{kt}$, where C and k are constants. Show that the rate at which $N(t)$ changes is directly proportional to $N(t)$.

14.) Use the limit definition of a derivative, $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$, to compute each of the following limits.

a.) $\lim\limits_{h\to 0} \dfrac{2^{3(1+h)} - 8}{h}$

b.) $\lim\limits_{h\to 0} \dfrac{5^{(h-1)} - (1/5)}{h}$

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The following problem is for recreational purposes only.

15.) A horse is tethered by a rope to the corner of a small shed with a square 10 ft. by 10 ft. floor. If the rope is 40 feet long, sketch the shape of the horse's grazing area. How close can you plant flowers to the shed and keep the horse from eating them ?

Thanks to Dr. Kouba