MAT 17A - DISCUSSION #7

November 10, 2015

Problem 1. Carbon Dating

The age of a fossil can be approximated by measuring the amount of ^{14}C in the fossil and comparing against the amount found in a typical living organism. It takes approximately 5,700 years for the amount of ^{14}C to decay to half it's original amount. Given that this decay is exponential, we can model it using an exponential equation with the form $Q(t) = c(b)^{kt}$.

(a) Find the equation, $Q(t)$, that models the decay in amount of ^{14}C. Use Q_0 for the initial quantity of ^{14}C.

(b) Solve $Q(t)$ for t to express t as a function of Q.

(c) What is the interpretation of $t(Q)$ in terms of "time" and "quantity of ^{14}C"? If you were studying a fossil, which function would you actually be using, $Q(t)$ or $t(Q)$, and why?

(d) Find dt/dQ. What is the interpretation of dt/dQ?

(e) Calculate the age of a fossil if it has the following percentage of its original ^{14}C.

i) 90%

ii) 80%

iii) 20%

iv) 10%

(f) Explain the difference between the jump from i) to ii) and from iii) to iv).

Problem 2. Exponential Depletion of Resources

In economist Thomas Malthus's 1798 paper, *An Essay on the Principle of Population*, he proposed an exponential growth model for the U.S. population and a linear growth model for food production. Let's take his model to say that the U.S. population would contain $N(t) = 8.3(1.33)^t$ million individuals t decades after 1815. Suppose the amount of food produced each year, measured in terms of rations (i.e. the amount of food needed to sustain one individual for one year), grew linearly during this same period with the amount given by the equation

$$R(t) = 10 + 4t$$

The number of surplus rations $S(t)$ over this period can be found by taking the difference of the above two functions

$$\begin{aligned} S(t) &= R(t) - N(t) \\ &= 10 + 4t - 8.3(1.33)^t \end{aligned}$$

(a) Plot $S(t)$ in RStudio and look at the point where S starts decreasing. What do you notice about the sha What can you say about the surplus of food in the United States?

(b) Plot $S'(t)$ in RStudio. Use the graph to determine at what point in time $S(t)$ starts decreasing and interpret what this means.

(c) Based on this model, when would the U.S. have entered a food shortage? Historically speaking, why might this not have actually happened? Which of the two models would have to be changed later in the 19^{th} century, and why?

Problem 3. Systolic Blood Pressure

The human heart goes through cycles of contraction and relaxation (called systoles). During these cycles, blood pressure goes up and down repeatedly as the heart contracts, pressure rises, and as the heart relaxes (for a split second), the pressure drops. Consider the following approximate function for the blood pressure of a patient:

$$P(t) = 100 + 20\cos\left(\frac{\pi t}{35}\right) \text{ mmHg}$$

where t is measured in minutes.

a) Find and interpret $P'(t)$. Plot $P(t)$ and $P'(t)$ on the same plot in RStudio.

b) Plot $P''(t)$, $P'''(t)$, and $P''''(t)$ on the same graph as in a). How are the graphs similar? How do they differ?

c) **Differential Equations**: Find the appropriate constants k, c so that

$$P''(t) = k\Big(P(t) + c\Big)$$

Problem 4. Height of Beer Froth

After pouring a mug full of the German beer Erdinger Weissbier, German physicist Leike measured the height of the beer froth at regular time intervals. He estimated the height (in centimeters) of the beer froth as

$$H(t) = 1.7(0.94)^t$$

where t is measured in seconds.

a) Find

$$\left.\frac{dH}{dt}\right|_{t=25}$$

and interpret this quantity.

b) **Differential Equations**: Find the appropriate constant k so that $H'(t) = kH(t)$.

Problem 5. Fermi Problem

If your life earnings were doled out to you at a certain rate per hour for every hour of your life, how much is your time worth?

a) Without the internet, use your intuition and modeling prowess to guess.

b) Google it and see how your guess compares.