MAT 17A - DISCUSSION #8

November 17, 2015

Problem 1. Approximation of $g(x) = \sqrt{x}$ and $h(x) = x^{\frac{3}{2}}$

Consider the functions $g(x) = \sqrt{x}$ and $h(x) = x^{\frac{3}{2}}$. We can approximate these functions near a real number $x = a$ using the **linear approximation of f around a**:

$$f(x) \approx f(a) + f'(a)(x - a)$$

a) State the linear approximations of g and h at $x = 9$.

b) Use the linear approximations from a) to approximate $\sqrt{10}$ and $10^{\frac{3}{2}}$. Compare this to the actual value of $\sqrt{10}$.

c) Complete the same process from b) for $\sqrt{20}, 20^{\frac{3}{2}}$ and $\sqrt{40}, 40^{\frac{3}{2}}$.

d) Use RStudio to plot g and its linear approximation around 9 for the domains $[9, 10]$, $[9, 20]$ and $[9, 40]$. Remember that you can plot two functions simultaneously using the commands:

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plot(x,g)
lines(x,gl)
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e) Use RStudio to plot h and its linear approximation around 9 for the domains $[9, 10]$, $[9, 20]$ and $[9, 40]$.

f) Consider $g''(a)$ and $h''(a)$ for any $a > 0$. Besides the magnitude of these values, what is one main difference between them? Consider the shapes of the graphs of $g(x)$ and $h(x)$, what is one main difference between them? Write down how you think these differences you have observed influence each other.

g) Are there any values a around which we cannot linearize g? How about h?

Problem 2. Metabolic Rate for CA Condor

In modeling problems, an error of measurement for our independent variable x can propagate to an error in estimation of the dependent variable y. This phenomenon know as **error propagation**, can be estimated using linear approximation. Use the linear approximation of $f(x)$ at $x = a$ to fill in the following lines:

$$\begin{aligned} \Delta y &= f(a + \Delta x) - f(a) \\ &\approx \underline{\hspace{2cm}} - f(a) \\ &= \underline{\hspace{2cm}} \end{aligned}$$

Note that in this particular context, the term $f'(a)$ in the final expression is referred to as the **sensitivity** of y to x at $x = a$.

a) Conceptually, why do you think we refer this term as **sensitivity**? What would a larger (or smaller) sensitivity value change about our linear approximation at a particular $x = a$?

To examine this phenomenon, let's consider a model for the metabolic rate of animals. The following curve models how the metabolic rate R (in kilocalories/day) depends on body mass M (in kilograms):

$$R = e^{4.2} M^{0.75}$$

b) Estimate the metabolic rate of a California condor weighing 10kg.

c) Given that 10kg is an estimate of the weight of a condor, what is the sensitivity of our model to this measurement? Select a small error ΔM and see how it propagates to an error ΔR in our estimate of R.

d) What are the units of this error propagation?

Absolute Error vs Percent Error

Note that in parts b-d above we were discussing absolute error Δy in our model. Often in science we would prefer to measure error relative, which can be done using $\frac{\Delta x}{a}$ in place of Δx.

Thus we define **elasticity** of a model f with respect to x nearby a with

$$E = f'(a)\frac{a}{f(a)}$$

e) Find the elasticity of our estimate of the metabolic rate to the estimate of the condor's weight.

f) Given a 10% error in the estimate of the condor's weight, what (approximate) error will we have in the metabolic rate?

g) What are the unit(s) of $\frac{\Delta M}{10kg}$ vs ΔM? What will the unit(s) be for E in general?

Problem 3. Pulse Rate and Error Propagation

In a healthy person of height h inches, the average pulse rate in beats per minute is modeled by the formula:

$$P(h) = \frac{596}{\sqrt{h}} \qquad\qquad 30 \leq h \leq 100$$

a) Compute the sensitivity of P at $h = 60$.

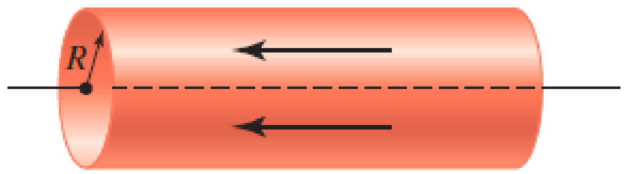
b) Use part a) to estimate the change in pulse rate that corresponds to a height change from 59 to 60 inches.

c) Compute the elasticity of P. Does it depend on h?

d) Determine how accurate the measurement of h needs to be to ensure the estimate for P has an error of less than 10%.

Problem 4. Blood Flow

When we consider the flow of blood through a blood vessel, such as a vein or artery, we can model the shape of the blood vessel by a cylindrical tube with radius R and length l.



Because of friction at the walls of the tube, the velocity v of the blood is greatest along the central axis of the tube and decreases as the distance r from the axis increase until v becomes 0 at the wall. In 1840, the French physician Jean-Louis-Marie Poiseuille discovered the relationship between v and r, called the **law of laminar flow**, which states:

$$v = \frac{P}{4\eta l}(R^2 - r^2)$$

where η is the viscosity of the blood and P is the pressure difference between ends of the tube. If P and l are constant, then v is a function of r with domain $[0, R]$.

(a) State the function for the blood flow velocity in a small human artery with a radius of 0.008cm, length of 2cm, pressure difference of 4000 dynes/cm^2 and blood viscosity 3.7×10^{-3} pascal-seconds (be VERY careful with units!). Use this function to find dv/dr. What are the units of dv/dr? What is the interpretation of this function?

(b) In general, if $y = f(x)$ then one way of writing a linear approximation is $\Delta y \approx f'(x)\Delta x$. What is the interpretation of this statement?

(c) Use Poiseuille's law of laminar flow to find the velocity at the central axis of a blood vessel (recall what the values R and r represent). If the radius changes, use $\Delta y \approx f'(x)\Delta x$ to explain how the relative change in the blood velocity is related to the relative change in the radius.

(d) From (c), if the radius of the artery is changed by 10%, what happens to the velocity in terms of percentage change?

(e) Another law of Poiseuille says that when blood flows along a blood vessel, the flux F (the volume of blood per unit time that flows past a given point) is proportional to the fourth power of the radius R of the blood vessel:

$$F = kR^4$$

A partially clogged artery can be expanded by an operation called angioplasty, in which a balloon-tipped catheter is inflated inside the artery in order to widen it. Show that the relative change in flux is about four times the relative change in radius. How will a 5% increase in the radius affect the flow of blood?

Problem 5. Fermi Problem

How many dump truck loads would it take to cart away Mt. Everest?

a) Without the internet, use your intuition and modeling prowess to guess.

b) Google relevant information and see how your guess compares.