Math 17A Vogler Discussion Sheet 9

1.) Assume that the derivative of y = f(x) is  $f'(x) = x^2(5-x)^3$ . Determine the x-values for inflection points on the graph of f.

2.) Do detailed graphing (See instruction sheet from class.) for each function .

a.)  $y = \sin x + \cos x$  on the interval  $[0, 2\pi]$ b.)  $y = x(x-4)^3$ c.)  $f(x) = \frac{x^2}{x^2 - 4}$ d.)  $f(x) = 3x^{1/3} - x$ 

3.) An open cylindrical can is to hold  $64\pi$  in.<sup>3</sup> What radius, r, and height, h, will require the least amount of material?

4.) A farmer has 600 ft. of fencing to construct a rectangular pigpen divided into four equalsized, parallel, rectangular sections. What dimensions will result in the largest possible total area of the pigpen ?

5.) A hiker is 6 miles directly west of a North-South road and her cabin is 10 miles North of the point on the road nearset to her. If she can walk at 4 mph off the road and at 5 mph on the road, find the least amount of time for her to reach the cabin.

6.) Find the dimensions of the rectangle of largest area which can be inscribed in a circle of radius 6.

7.) Determine the length of the shortest ladder which will reach over an 8-ft. high fence to a large wall which is 3 ft. behind the fence.



8.) There are 50 apple trees in an orchard. Each tree produces 800 apples. For each additional tree planted in the orchard, the output per tree drops by 10 apples. How many trees should be added to the existing orchard in order to maximize the total output of

apples ?

9.) Find the point (x, y) on the graph of  $y = \sqrt{x}$  which is nearest the point (4, 0).

10.) Find the point P = (x, 0) on the x-axis which minimizes the sum of distances from (0, 4) to P and from P to (3, 2).

1.) Use any method to determine the following limits.

a.) 
$$\lim_{x \to 3} \frac{x^2 - 3x}{x^2 - 9}$$
 b.)  $\lim_{x \to \infty} \frac{\ln x}{x + \ln x}$  c.)  $\lim_{x \to 0} \frac{\tan x^2}{x \sin x}$   
d.)  $\lim_{n \to \infty} \left(1 + \frac{5}{n}\right)^n$  e.)  $\lim_{x \to \infty} \left(\ln x\right)^{1/x}$  f.)  $\lim_{x \to 0^+} \frac{e^x - 1}{\sqrt{x}}$   
g.)  $\lim_{x \to \infty} \frac{e^x - 1}{\sqrt{x}}$  h.)  $\lim_{x \to 0} \frac{(e^x - 1)^2}{x^2}$  i.)  $\lim_{x \to 0^+} x^{\tan x}$   
j.)  $\lim_{x \to 0} \frac{e^{-1/x^2}}{x}$  k.)  $\lim_{n \to \infty} (3^n + 4^n)^{1/n}$  l.)  $\lim_{x \to 0} \sin x \ln x$ 

2.) Use the Intermediate Value Theorem to prove that each of the following equations is solvable. Then use the Newton-Raphson Method to find each solution to three decimal places.

a.) 
$$x^3 - x + 2$$
 b.)  $e^x + x - 3 = 0$ 

3.) a.) Prove that  $\log_B x = \frac{\ln x}{\ln B}$ . b.) Assume that 0 < B < 1 and compute i.)  $\lim_{x \to \infty} \log_B x$  ii.)  $\lim_{x \to 0^+} \log_B x$  iii.)  $\lim_{x \to \infty} \log_{Bx} x$ 

The following problem is for recreational purposes only.

11.) Two bicyclists are twelve miles apart. They begin riding toward each other, one pedaling at 4 mph and the other at 2 mph. At the same time a bumblebee begins flying back and forth between the riders at a constant speed of 10 mph. What is the total distance the bumblebee travels by the time the riders meet ?