1.) Determine the following limits.

a.) $\lim_{x\to 1} \frac{x^3-1}{x^2-1} \overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 1} \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)} = \frac{3}{2}$

b.) $\lim_{x\to 0} \frac{\sqrt{3+x}-\sqrt{3-x}}{x} \cdot \frac{\sqrt{3+x}+\sqrt{3-x}}{\sqrt{3+x}+\sqrt{3-x}}$

$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{(3+x)-(3-x)}{x(\sqrt{3+x}+\sqrt{3-x})} = \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}(\sqrt{3+x}+\sqrt{3-x})}$

$= \frac{2}{\sqrt{3}+\sqrt{3}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

c.) $\lim_{x\to 2} \frac{\frac{1}{x-1}-\frac{1}{5-2x}}{\frac{x-2}{1}} \overset{\frac{0}{0}}{=} \lim_{x\to 2} \frac{(5-2x)-(x-1)}{(x-1)(5-2x)} \cdot \frac{1}{x-2}$

$= \lim_{x\to 2} \frac{5-2x-x+1}{(x-1)(5-2x)(x-2)} = \lim_{x\to 2} \frac{6-3x}{(x-2)(5-2x)(x-2)}$

$= \lim_{x\to 2} \frac{-3\cancel{(x-2)}}{(x-1)(5-2x)\cancel{(x-2)}} = \frac{-3}{1} = -3$

d.) $\lim_{x\to\infty} \frac{2x^2+5}{7x^3-4} \cdot \frac{1/x^3}{1/x^3} \overset{\text{"}\frac{\infty}{\infty}\text{"}}{=} \lim_{x\to\infty} \frac{2/x + 5/x^3}{7-4/x^3}$

$= \frac{0+0}{7-0} = \frac{0}{7} = 0$

2.) a.) Determine the domain of $f(x) = \sqrt{7-x}$.

$7 - X \geq 0 \rightarrow 7 \geq X$ so

Domain: all $X \leq 7$

b.) Determine the range of $f(x) = 3 + 5\sin x$.

$-1 \leq \sin X \leq +1 \rightarrow -5 \leq 5\sin X \leq +5 \rightarrow$

$-2 \leq 3 + 5\sin X \leq 8$ so

Range: $-2 \leq Y \leq 8$

5. (*pts total*) Let $X = \log x$ and $Y = \log y$ for the following problems

(a) (*pts*) When the following table data is graphed on a log-log plot (i.e. using X and Y coordinates), a straight line results. Determine the equation of this line and graph the resulting line on a log-log plot.

x	X	y	Y
1	0	10000	4
10	1	100	2

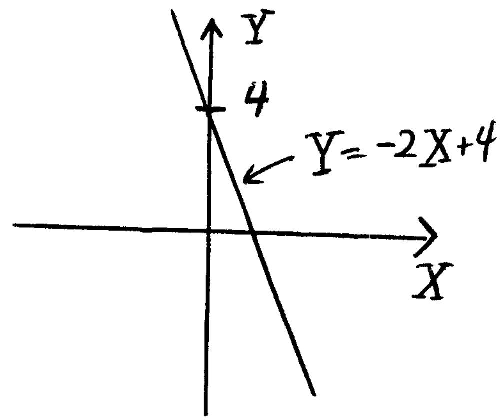
Using points $(0,4)$ & $(1,2)$,

$m = \dfrac{Y_1 - Y_2}{X_1 - X_2} = \dfrac{4-2}{0-1} = -2,$

Y-int: $b = 4 = \log 10{,}000$

$\Rightarrow Y = mX + b = -2X + 4$

$\Rightarrow Y = -2X + 4$



(b) (*pts*) Assume in part (a) your equation of a line was $Y = -4X + 1$, use the appropriate logarithmic transformation to find the function relationship between x and y.

$Y = -4X + 1 \Rightarrow \log y = -4\log x + \log 10 = \log x^{-4} + \log 10$

$\Rightarrow \log y = \log 10x^{-4} \Rightarrow \boxed{y = 10x^{-4}}$

4.) Consider the following function $f(x) = \begin{cases} \dfrac{x^2 - 3x}{x^2 - 9}, & \text{if } x \neq 3, -3 \\ \frac{1}{2}, & \text{if } x = 3 \\ 0, & \text{if } x = -3. \end{cases}$

Determine if f is continuous at $x = 3$.

i.) $f(3) = 1/2$

ii.) $\lim_{x \to 3} f(x) = \lim_{x \to 3} \dfrac{x^2 - 3x}{x^2 - 9} \overset{\text{"}0/0\text{"}}{=} \lim_{x \to 3} \dfrac{x\cancel{(x-3)}}{\cancel{(x-3)}(x+3)}$

$= 3/6 = 1/2$

iii.) $\lim_{x \to 3} f(x) = f(3)$ so

f is continuous at $x = 3$.

5.) Determine all possible fixed points for the following recursion : $a_{n+1} = \dfrac{3a_n^2}{a_n^2 - 4}$

$\rightarrow L = \dfrac{3L^2}{L^2 - 4} \rightarrow L(L^2 - 4) = 3L^2$

$\rightarrow L^3 - 4L - 3L^2 = 0$

$\rightarrow L^3 - 3L^2 - 4L = 0$

$\rightarrow L(L^2 - 3L - 4) = 0$

$\rightarrow L(L-4)(L+1) = 0$

$L = 0 \quad L = 4 \quad L = -1$

6.) Consider the function $f(x) = \dfrac{x}{3-x}$.

a.) Show algebraically that f is one-to-one.

assume $f(x_1) = f(x_2)$

$$\rightarrow \frac{x_1}{3-x_1} = \frac{x_2}{3-x_2}$$

$$\rightarrow 3x_1 - \cancel{x_1 x_2} = 3x_2 - \cancel{x_1 x_2}$$

$$\rightarrow 3x_1 = 3x_2 \longrightarrow x_1 = x_2$$

b.) Determine $y = f^{-1}(x)$, the inverse function for $y = f(x)$.

$Y = \dfrac{X}{3-X} \rightarrow$ (switch variables)

$\rightarrow X = \dfrac{Y}{3-Y} \rightarrow$ (solve for Y)

$$\rightarrow 3X - XY = Y \rightarrow 3X = XY + Y$$

$$\rightarrow 3X = Y(X+1) \rightarrow Y = \frac{3X}{X+1}$$

$$\rightarrow f^{-1}(x) = \frac{3X}{X+1}$$

7.) Find a formula for the nth term (starting with n=0) of each of the following sequences.

a.) $\frac{7}{2}, \frac{4}{4}, \frac{1}{8}, \frac{-2}{16}, \frac{-5}{32}, \frac{-8}{64}, \ldots$

$n: \quad 0, \quad 1, \quad 2, \quad 3, \quad \ldots, \quad n$

$a_n: \quad \frac{7}{2}, \quad \frac{7-3}{2^2}, \quad \frac{7-6}{2^3}, \quad \frac{7-9}{2^4}, \quad \ldots, \quad \boxed{\frac{7-3n}{2^{n+1}}}$

b.) $5, 7, 10, 14, 19, 25, 32, \cdots$
(HINT: Use the fact that $1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$.)

n	a_n
0	$5 = (1) + 4$
1	$7 = (1+2) + 4$
2	$10 = (1+2+3) + 4$
3	$14 = (1+2+3+4) + 4$
4	$19 = (1+2+3+4+5) + 4$
$\vdots$	$\vdots$
n	$(1+2+3+\cdots+(n+1)) + 4$

$$= \frac{(n+1)((n+1)+1)}{2} + 4$$

$$= \frac{1}{2}(n+1)(n+2) + 4$$

8.) Use the Squeeze Principle (Sandwich Theorem) to determine the limit of the following sequence :

$$a_n = \frac{n + 3\sin n}{n+3}$$

$$-1 \leq \sin n \leq +1$$

$$\rightarrow -3 \leq 3 \sin n \leq 3$$

$$\rightarrow n - 3 \leq n + 3\sin n \leq n + 3$$

$$\rightarrow \frac{n-3}{n+3} \leq \frac{n + 3\sin n}{n+3} \leq \frac{n+3}{n+3} = 1$$

and $\lim_{n\to\infty} \frac{n-3}{n+3} \cdot \frac{\frac{1}{n}}{\frac{1}{n}} \overset{\frac{\infty}{\infty}}{=} \lim_{n\to\infty} \frac{1 - 3/n}{1 + 3/n} = \frac{1-0}{1+0} = 1$,

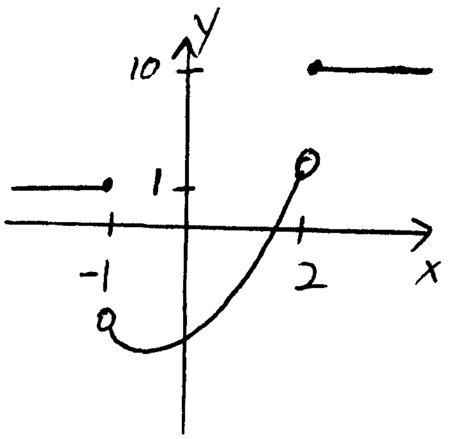
$\lim_{n\to\infty} 1 = 1$, so by Squeeze Principle

$$\lim_{n\to\infty} \frac{n + 3\sin n}{n+3} = 1$$

(b) Consider the following function

$$f(x) = \begin{cases} 1 & \text{if } x \le -1 \\ Ax^2 + Bx & \text{if } -1 < x < 2 \\ 10 & \text{if } x \ge 2 \end{cases}$$

Use limits and a "fake graph" to determine the value of constants A and B so that the following function is continuous for all values of x .



We need for continuity ('kiss condition')

$$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x) \ \& \ \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$\Rightarrow 1 = \lim_{x\to -1^+} Ax^2+Bx \ \& \ \lim_{x\to 2^-} Ax^2+Bx = 10$$

$$\Rightarrow A-B=1 \ \& \ 4A+2B=10$$

$$\Rightarrow \underline{A=B+1} \ \& \ \underline{2A+B=5}$$

sub

$$\Rightarrow 2(B+1)+B=5 \Rightarrow 3B+2=5 \Rightarrow 3B=3$$

$$\Rightarrow \boxed{B=1 \ \& \ A=1+1=2}$$

The following EXTRA CREDIT PROBLEM is worth . This problem is OPTIONAL.

1.) Determine the next three numbers in the following sequence :
$-2, 0, 0, 4, 18, 48, 100, \cdots$

n: 0 , 1 , 2 , 3 , 4 , 5 , 6

a_n: $-2\cdot 1$, $-1\cdot 0$, $0\cdot 1$, $1\cdot 4$, $2\cdot 9$, $3\cdot 16$, $4\cdot 25$

a_n: $-2(-1)^2$, $-1\cdot(0)^2$, $0\cdot 1^2$, $1\cdot 2^2$, $2\cdot 3^2$, $3\cdot 4^2$, $4\cdot 5^2$

for n → $a_n = (n-2)(n-1)^2$ for $n = 0, 1, 2, \ldots$

so next three numbers are

$5\cdot 6^2 = 180$, $6\cdot 7^2 = 294$, and $7\cdot 8^2 = 448$