1.) Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

a.) $y = x^{3/4} + \sqrt{123} - 2x^{-7}$

$$\xrightarrow{D} \quad Y' = \frac{3}{4}X^{-1/4} + 0 - 2 \cdot -7X^{-8}$$

b.) $f(x) = \dfrac{2^x}{6 + e^{3x}}$

$$\xrightarrow{D} \quad f'(x) = \frac{(6+e^{3x}) \cdot 2^x \ln 2 - 2^x \cdot e^{3x} \cdot 3}{(6+e^{3x})^2}$$

d.) $f(x) = (\ln x)^x$ $\longrightarrow \ln f(x) = \ln (\ln x)^x = x \cdot \ln(\ln x) \xrightarrow{D}$

$$\frac{1}{f(x)} \cdot f'(x) = \cancel{x} \cdot \frac{1}{\ln x} \cdot \frac{1}{\cancel{x}} + (1) \cdot \ln(\ln x) \longrightarrow$$

$$f'(x) = (\ln x)^x \cdot \left\{ \frac{1}{\ln x} + \ln(\ln x) \right\}$$

2.) Let $f(x) = x(5 - x)^4$.

a.) Solve $f'(x) = 0$ for x .

$$\xrightarrow{D} \quad f'(x) = x \cdot 4(5-x)^3 \cdot (-1) + (1)(5-x)^4$$
$$= (5-x)^3 \cdot [-4x + (5-x)]$$
$$= (5-x)^3 \cdot [5 - 5x] = 0 \rightarrow \quad x = 5, \ x = 1$$

b.) Solve $f''(x) = 0$ for x .

$$\xrightarrow{D} \quad f''(x) = (5-x)^3 [-5] + 3(5-x)^2(-1)[5-5x]$$
$$= -(5-x)^2 \cdot [5(5-x) + 3(5-5x)]$$
$$= -(5-x)^2 [25 - 5x + 15 - 15x]$$
$$= -(5-x)^2 [40 - 20x] = 0 \rightarrow \quad x = 5, \ x = 2$$

3.) Use the Intermediate Value Theorem to prove that the equation $x^3 = 4 - x$ is solvable. This is a writing exercise. You will be scored on proper style and mathematical correctness.

$x^3 = 4 - x \rightarrow x^3 + x - 4 = 0$ so let $f(x) = x^3 + x - 4$ and $m = 0$. Function f is continuous for all x-values since f is a polynomial. and $f(1) = -2$, $f(2) = 6$, and $m = 0$ is between $f(1)$ and $f(2)$. Choose interval $[1, 2]$. Thus, by IMVT there is a # c, $1 \le c \le 2$, so that $f(c) = m$, i.e., $c^3 + c - 4 = 0$ and the equation is solvable.

5. Use linearization to estimate the value of $\sqrt{8}$. DO NOT SIMPLIFY THE ANSWER.

Let $f(x) = \sqrt{x}$ & $a = 9$

$\Rightarrow f'(x) = \frac{1}{2\sqrt{x}}$

$L(x) = f(a) + f'(a)(x-a)$

$= \sqrt{9} + \frac{1}{2\sqrt{9}}(x-9) = 3 + \frac{1}{6}(x-9)$

$\Rightarrow L(x) = 3 + \frac{1}{6}(x-9)$

$\sqrt{8} \approx L(8) = 3 + \frac{1}{6}(-1)$ or $2\frac{5}{6}$

5.) Assume that y is a function of x and $xy^2 + y = x + 3$. Determine an equation of the line perpendicular to this graph at $x = 0$.

If $X=0$, then $(0)Y^2 + Y = 0 + 3 \rightarrow Y = 3$;

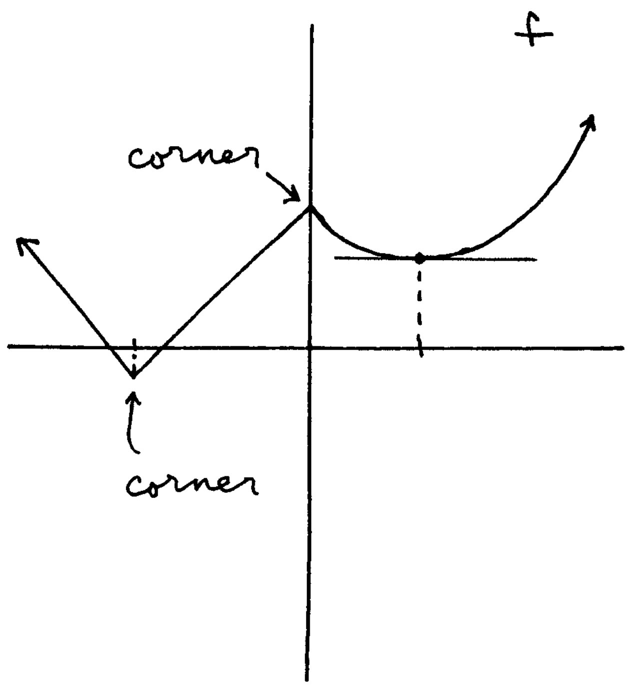
$XY^2 + Y = X + 3 \xrightarrow{D} X \cdot 2YY' + (1)Y^2 + Y' = 1 + 0 \rightarrow$

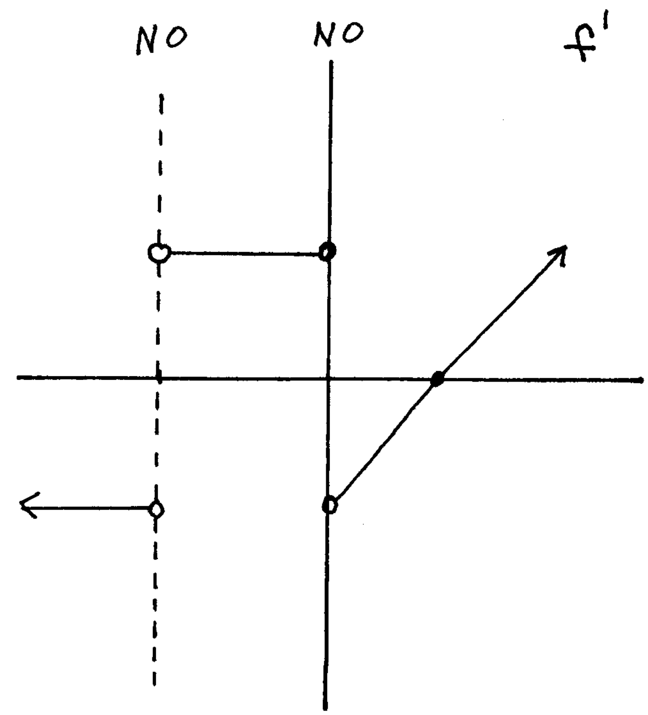
$Y'(2XY+1) = 1 - Y^2 \rightarrow Y' = \dfrac{1-Y^2}{2XY+1}$ and

$X=0, Y=3 \rightarrow Y' = \dfrac{1-9}{2(0)(3)+1} = -8$ so $\perp$ slope is $m = \frac{1}{8}$ and line is

$$Y - 3 = \frac{1}{8}(X - 0) \text{ or } Y = \frac{1}{8}X + 3$$

6.) Sketch a graph of the derivative f' using the given graph of f.





6.

(a) State the definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

(b) Use the definition of the derivative to differentiate the function

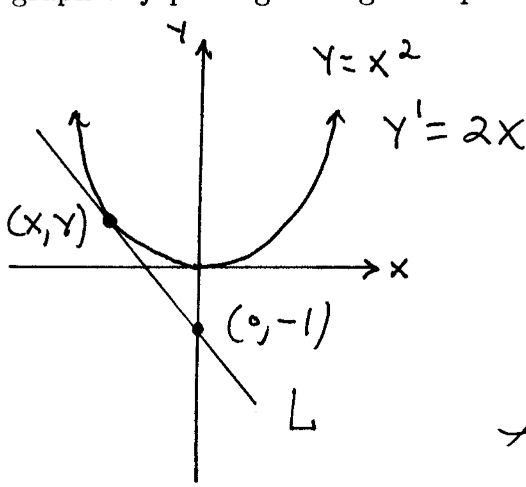
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(h)}{h} = \lim_{h \to 0} \frac{\frac{x+h}{x+h+7} - \frac{x}{x+7}}{h} \cdot \frac{(x+7)(x+h+7)}{(x+7)(x+h+7)}$$

$$= \lim_{h \to 0} \frac{(x+h)(x+7) - x(x+h+7)}{h(x+7)(x+h+7)}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + \cancel{7x} + \cancel{hx} + 7h - \cancel{x^2} - \cancel{hx} - \cancel{7x}}{h(x+7)(x+h+7)}$$

$$= \lim_{h \to 0} \frac{7\cancel{h}}{\cancel{h}(x+7)(x+h+7)} = \frac{7}{(x+7)^2}$$

9.) Find all points (x, y) on the graph of $f(x) = x^2$ with tangent lines to the graph of f passing through the point $(0, -1)$.



SLOPE of line L is

1. $m = 2X$

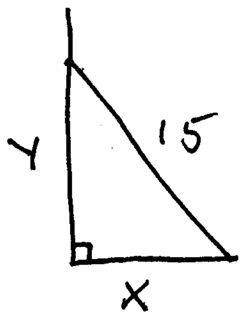
2. $m = \frac{Y-(-1)}{X-0} = \frac{Y+1}{X} = \frac{X^2+1}{X}$;

then set equal →

$$2X = \frac{X^2+1}{X} \rightarrow 2X^2 = X^2 + 1 \rightarrow X^2 = 1 \rightarrow X = \pm 1 \rightarrow$$

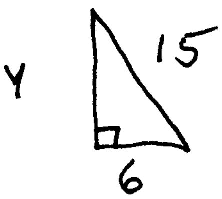
$X = 1,\ Y = 1$ or $X = -1,\ Y = 1$

8.) A 15-foot ladder is leaning against a wall. If the base of the ladder is pushed toward the wall at the rate of 2 ft./sec., at what rate is the top of the ladder moving up the wall when the base of the ladder is 6 ft. from the wall ?



assume $\frac{dX}{dt} = -2$ ft./sec.; find $\frac{dY}{dt}$ when $X = 6$ ft. :

$$X^2 + Y^2 = 15^2 \xrightarrow{D} 2X \cdot \frac{dX}{dt} + 2Y \frac{dY}{dt} = 0$$



$6^2 + Y^2 = 15^2 \to Y = \sqrt{189} \approx 13.7$

$$(6)(-2) + \sqrt{189} \cdot \frac{dY}{dt} = 0 \to$$

$$\frac{dY}{dt} = \frac{12}{\sqrt{189}} \approx 0.87 \text{ ft./sec.}$$

3.) You are standing on the top edge of a building which is 96 ft. high. You throw an apple straight UP at 80 ft./sec. and watch as it falls back to the ground.

a.) Assume that the acceleration due to gravity is $s''(t) = -32$ ft./sec.^{2} . Derive velocity, $s'(t)$, and height (above ground), $s(t)$, formulas for this apple.

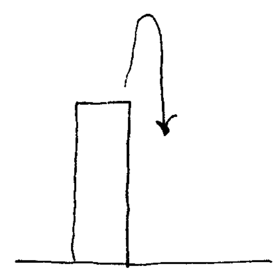
$s''(t) = -32 \to$

$s'(t) = -32t + C$ (and $s'(0) = 80$ ft./sec.)

$\to C = 80 \to \boxed{s'(t) = -32t + 80}$;

$s(t) = -16t^2 + 80t + C$ (and $s(0) = 96$ ft.)

$\to C = 96 \to \boxed{s(t) = -16t^2 + 80t + 96}$



b.) In how many seconds will the apple strike the ground ?

strike ground : $s(t) = 0 \to$

$-16t^2 + 80t + 96 = 0 \to -16(t^2 - 5t - 6) = 0 \to$

$-16(t-6)(t+1) = 0 \to t = 6$ sec.

c.) How high does the apple go ?

highest point: $s'(t) = 0 \to -32t + 80 = 0 \to$

$\boxed{t = 2.5 \text{ sec.}} \to$

$s(2.5) = -16(2.5)^2 + 80(2.5) + 96 = 196$ ft.

The following EXTRA CREDIT PROBLEM is worth . This problem is OPTIONAL.

1.) Evaluate the following limit : $\lim_{x\to 0} \frac{\sin x^2 \cdot \sin^2(\sin x^2)}{\cos^2 x^2 - 1}$

$$\overset{\text{"}\frac{0}{0}\text{"}}{=} \lim_{x\to 0} \frac{\sin x^2 \cdot \sin(\sin x^2) \cdot \sin(\sin x^2)}{-\sin^2 x^2}$$

$$= \lim_{x\to 0} \frac{-\cancel{\sin} x^2}{\cancel{\sin} x^2} \cdot \frac{\sin(\sin x^2)}{\sin x^2} \cdot \sin(\sin x^2)$$

$$= (-1) \cdot (1) \cdot \sin(\sin 0)$$

$$= -1 \sin 0$$

$$= -1 (0) = 0$$