1.) Differentiate each of the following functions. DO NOT SIMPLIFY ANSWERS.

a.) $y = \sin^5(3 - x)$

b.) $f(x) = \dfrac{2^x}{6 + e^{3x}}$

c.) $f(x) = (\ln x)^3 \cdot \log_4(\tan x)$

d.) $f(x) = (\ln x)^x$

5.) Consider the function $f(x) = x - \sqrt{x}$ defined on the closed interval $[0, 4]$. Verify that f satisfies the assumptions of the Mean Value Theorem (MVT) and find all values of c guaranteed by the MVT.

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3.) The manager of the Economy Motel charges \$30 per room and rents ~~50~~ rooms each night. For each \$5 increase in room charge four (4) fewer rooms are rented. What charge per room will maximize the total amount of money the manager will make in one night ?

4.) Use the limit definition of derivative to differentiate $f(x) = \dfrac{x^2}{x+1}$.

5.) Use limits to determine the values of the constants A and B so that the following function is continuous for all values of x.

$$f(x) = \begin{cases} Bx^2 + Ax, & \text{if } x \le -1 \\ 2B - Ax, & \text{if } -1 < x \le 2 \\ x + 3, & \text{if } x > 2 . \end{cases}$$

7.) **Use a linearization to estimate the value of $\sqrt{68}$.**

7.) The radius of a sphere is measured with an absolute percentage error of at most 4% . Use differentials to estimate the maximum absolute percentage error in computing the volume of the sphere. ($V = \frac{4}{3}\pi r^3$.)

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8. Use the Intermediate Value Theorem to show that the equation $x^3 + x = \sqrt{x+4}$ is solvable. This is a writing exercise.

9.) Car B is 34 miles directly east of car A and begins moving west at 90 mph. At the same moment car A begins moving north at 60 mph.

a.) At what rate is the distance between the cars changing after t $= \frac{1}{5}$ hr. ?

9.) Consider the Beverton-Holt Recursion given by $N_{t+1} = \dfrac{120N_t}{100 + N_t}$ with initial amount $N_0 = 10$.

a.) Find all fixed points for this recursion.

b.) Determine the growth parameter R and the carrying capacity K.

c.) Find N_1.

10.) Find the slope and concavity of the graph $xy + y^2 = 3x + 1$ at the point $(0, -1)$.

11.) Consider all rectangles in the first quadrant inscribed in such a way that their bases lie on the x-axis with the top corner on the graph of $y = \sqrt{4 - x}$. Find the length and width of the rectangle of maximum area.

12.) Consider the function $f(x) = x\, e^{\left(\frac{-x}{2}\right)}$. Determine where f is increasing, decreasing, concave up, and concave down. Identify all relative and absolute extrema, inflection points, x- and y-intercepts, and vertical and horizontal asymptotes. Sketch the graph. You may assume that $f'(x) = (1 - \frac{x}{2})\, e^{\left(\frac{-x}{2}\right)}$ and $f''(x) = (\frac{x}{4} - 1)\, e^{\left(\frac{-x}{2}\right)}$.

4.) In 1947 earthenware jars containing what are known as the Dead Sea Scrolls were found. Analysis showed that the scrolls contained 76 % of their original carbon-14. Assuming the half-life of carbon-14 is 5730 years, estimate the age of the Dead Sea Scrolls when they were found.

14.) Evaluate the following limits.

a.) $\lim_{x \to 0} \dfrac{x \sin x}{(\arctan x)^2}$

b.) $\lim_{x \to 0^+} x \ln x$

c.) $\lim_{x \to \infty} (x^3 + 4)^{\frac{1}{x}}$

The following EXTRA CREDIT PROBLEM is worth 10 points. This problem is OPTIONAL.

1.) You are standing at point A on the edge of a river 1 mile wide. You are to get to point B, which is 4 miles from the point directly across the river from you. You can paddle a canoe in the water at a speed of 10 miles per hour and you can ride a bicycle on land at a speed of 15 miles per hour. Determine x so that the time it takes to go from point A to point B is a minimum and determine the minimum time.