Math 17A
Vogler
Carbon Dating

In 1960 the American scientist W. F. Libby won the Nobel prize for his discovery of carbon dating, a method for determining the age of certain fossils. Carbon dating is based on the fact that nitrogen is converted to radioactive carbon-14 by cosmic radiation in the upper atmosphere. This radioacative carbon is absorbed by plant and animal tissue through the life processes (for example, through respiration) while the plant or animal lives. However, when the plant or animal dies the absorption process stops and the amount of carbon-14 decreases (exponentially) through radioacative decay.

When the object, such as a piece of wood or bone, was part of a living organism, it accumulated small amounts of radioactive carbon-14, so that a certain proportion of the carbon in the object was carbon-14. Once the organism dies, it no longer picks up carbon-14 through interaction with its environment. By measuring the proportion of carbon-14 in the fossilized object, comparing that to the proportion in living material, and using the fact that the half-life of carbon-14 is about 5730 years, the age of the object can be estimated.

<u>EXAMPLE</u> : Assume that a portion of fossilized tree found today contains about 23.7% of its original amount of carbon-14. Estimate the age of the fossil.

Let N = amount of Carbon-14 @ time t (yrs)

We assume exponential growth

$$N = N_0 e^{kt}$$

Half-life of $t = 5730$ yrs $\Rightarrow N = \frac{1}{2}N_0$ when $t = 5730$

$$\Rightarrow \frac{1}{2}\cancel{N_0} = \cancel{N_0} e^{5730k} \Rightarrow \ln\frac{1}{2} = \cancel{\ln e}^{5730k} = 5730k$$

$$\Rightarrow k = \frac{\ln(\frac{1}{2})}{5730} \Rightarrow \boxed{N = N_0 e^{\frac{\ln(\frac{1}{2})}{5730}t}} \leftarrow \text{Exponential Growth Eqn for Carbon-14}$$

- If $N = 23.7\%$ of $N_0 = 0.237 N_0$

$$\Rightarrow 0.237 \cancel{N_0} = \cancel{N_0} e^{\frac{\ln(\frac{1}{2})}{5730}t} \Rightarrow \ln(0.237) = \cancel{\ln e}^{\frac{\ln(\frac{1}{2})}{5730}t}$$

$$\Rightarrow \ln(0.237) = \frac{\ln(\frac{1}{2})}{5730}t$$

$$\Rightarrow t = \frac{5730 \cdot \ln(0.237)}{\ln(\frac{1}{2})} \approx \boxed{11,901 \text{ yrs old}}$$