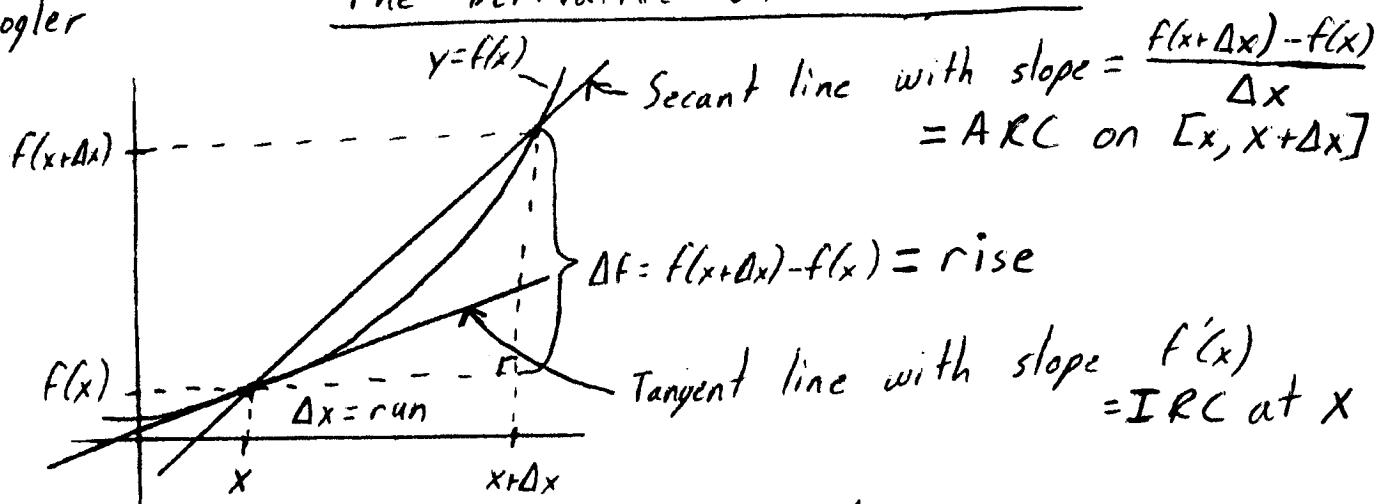


The Derivative of a Function



Note that the slope of the secant line is

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{f(x+h) - f(x)}{\Delta x}$$

We can obtain slope of tangent line by letting $h := \Delta x$ get 'small' (i.e. $\Delta x \rightarrow 0$) which leads to the following:

Defn The derivative of f at x is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Notes: 1) $f'(x)$ is the slope of tangent line through point $(x, f(x))$.

2) $\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \approx \frac{\Delta f}{\Delta x}$ can be considered 'infinitesimal division' (i.e. division by two 'small' numbers).

3) The derivative is sometimes referred to as the Instantaneous Rate of Change (IRC).

Defn The Average Rate of Change (ARC) of a function $y = f(x)$ on interval $[a, b]$ is

$$\text{A.R.C} = \frac{f(b) - f(a)}{b - a}$$

Note: ARC is slope of secant line between $(a, f(a))$ & $(b, f(b))$.