Math 17A
Vogler

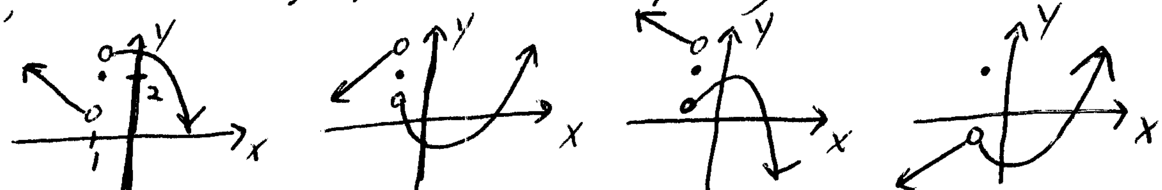
Forcing a Piece-wise Function to be Continuous

Ex Use limits to determine constants A & B such that the following function is continuous for all x-values

$$f(x) = \begin{cases} Ax^2 + B^2x - 2 & \text{if } x > -1 \\ 2 & \text{if } x = -1 \\ Bx - A + 8 & \text{if } x < -1 \end{cases}$$

Notice for any A, B $y = Ax^2 + B^2x - 2$ is a parabola & $y = Bx - A + 8$ is a line. So possible graphs of f, depending on A & B are:

'Fake Graphs'



Also, notice that $y = Ax^2 + B^2x - 2$ & $y = Bx - A + 8$ are polynomials, which are continuous for all values of x.

Hence, we need to force continuity @ $x = -1$. By conditions ii) & iii) of the definition of continuity, we must have

$$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^+} f(x) = \lim_{x \to -1} f(x) \quad \& \quad \lim_{x \to -1} f(x) = f(-1) = 2$$

or equivalently,

$$\lim_{x \to -1^-} f(x) = f(-1) = 2 \qquad \& \qquad \lim_{x \to -1^+} f(x) = f(-1) = 2$$

$\Rightarrow \lim_{x \to -1^-} Bx - A + 8 = 2$

$\Rightarrow -B - A + 8 = 2$

$\Rightarrow \boxed{A = B - 6}$ (1)

$\Rightarrow \lim_{x \to -1^+} Ax^2 + B^2x - 2 = 2$

$\Rightarrow (-1)^2A + B^2(-1) - 2 = 2$

$\Rightarrow A - B^2 - 2 = 2$

$\Rightarrow \boxed{A = B^2 + 4}$ (2)

(1)&(2) $\Rightarrow B^2 + 4 = 6 - B \Rightarrow B^2 + B - 2 = 0 \Rightarrow (B-1)(B+2) = 0$

$\Rightarrow \boxed{B = 1, A = 5 \quad \text{or} \quad B = -2, A = 8}$