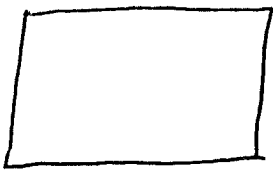
Math 17A
Vogler

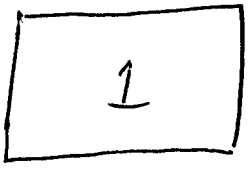
Sequences Example

Find the max number of rectangles formed by 200 vertical lines drawn through the given rectangle. Count rectangles of all sizes, including overlapping ones

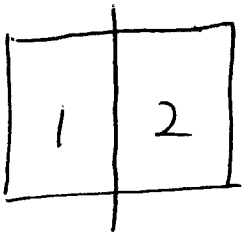


Note: When you add another vertical line, you still have all the old rectangles plus the new ones shown in the side figure

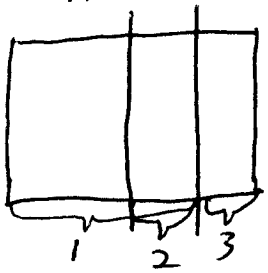
$n = 0$



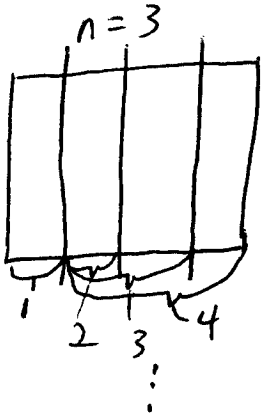
$n = 1$



$n = 2$



$n = 3$



lines (n)
0
1
2
3
4
⋮
n

of □'s (a_n)
$1 = 1$
$1+2 = 3$
$1+2+3 = 6$
$1+2+3+4 = 10$
$1+2+3+4+5 = 15$
⋮
$1+2+\dots+n+n+1$

$$\Rightarrow \boxed{a_n = \frac{(n+1)(n+2)}{2}}$$

For $n = 200$, $a_{200} = \dfrac{(201)(202)}{2} = \boxed{20{,}301 \text{ rectangles}}$