Section 4.1

1.) $f(x) = 5 \rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{5-5}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0,$

so $f'(1) = 0$

2.) $f(x) = -3x \rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{-3(x+h) - (-3x)}{h}$

$= \lim_{h \to 0} \frac{\cancel{-3x} - 3h + \cancel{3x}}{h} = \lim_{h \to 0} \frac{-3\cancel{h}}{\cancel{h}}$

$= \lim_{h \to 0} -3 = -3$ so $f'(-2) = -3$

5.) $f(x) = 2x^2 \rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{2(x+h)^2 - 2x^2}{h}$

$= \lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) - 2x^2}{h}$

$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4hx + 2h^2 - \cancel{2x^2}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h)}{\cancel{h}} = 4x + 2(0) \rightarrow$

$f'(x) = 4x$ so $f'(0) = 4(0) = 0$

10.) $f(x) = -x^2 + 4 \to f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{-(x+h)^2 + 4 - (-x^2 + 4)}{h}$

$= \lim_{h \to 0} \frac{-(x^2 + 2hx + h^2) + \cancel{4} + x^2 - \cancel{4}}{h}$

$= \lim_{h \to 0} \frac{\cancel{-x^2} - 2hx - h^2 \cancel{+x^2}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(-2x - h)}{\cancel{h}} = -2x$ so

$f'(x) = -2x$; $f'(x) = 0 \to -2x = 0 \to x = 0$

13.) $f(x) = x^2 - 6x + 9 \to f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - 6(x+h) + 9 - (x^2 - 6x + 9)}{h}$

$= \lim_{h \to 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{6x} - 6h + \cancel{9} \cancel{- x^2} + \cancel{6x} \cancel{- 9}}{h}$

$= \lim_{h \to 0} \frac{\cancel{h}(2x + h - 6)}{\cancel{h}} = 2x - 6$ so

$f'(x) = 2x - 6$; $f'(x) = 0 \to 2x - 6 = 0 \to$

$2x = 6 \to x = 3$.

Section 4.1

21.) a.) $f(x) = 5x^2 \longrightarrow f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{5(x+h)^2 - 5x^2}{h} = \lim_{h\to 0} \frac{5(x^2+2hx+h^2) - 5x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{5x^2} + 10hx + 5h^2 - \cancel{5x^2}}{h}$$

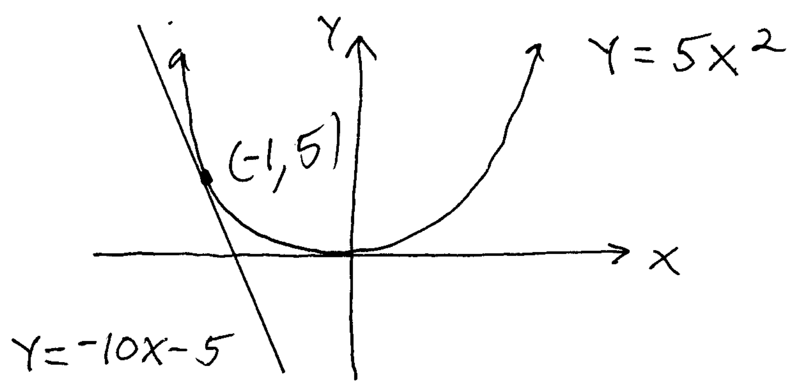
$$= \lim_{h\to 0} \frac{\cancel{h}(10x+5h)}{\cancel{h}} = 10x \text{, i.e.,}$$

$$f'(x) = 10x \longrightarrow f'(-1) = 10(-1) = -10$$

b.) $f(-1) = 5(-1)^2 = 5(1) = 5$ so $(-1, 5)$ is on the graph; slope of tangent line is $m = f'(-1) = -10$, so line is $y - 5 = -10(x-(-1)) \rightarrow$

$$y - 5 = -10x - 10 \longrightarrow y = -10x - 5$$

c.)



23.) a.) $f(x) = 1 - x^3 \longrightarrow f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{(1-(x+h)^3) - (1-x^3)}{h}$$

$$= \lim_{h\to 0} \frac{1 - (x^3 + 3x^2h + 3xh^2 + h^3) - 1 + x^3}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{1} - \cancel{x^3} - 3x^2h - 3xh^2 - h^3 - \cancel{1} + \cancel{x^3}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}(-3x^2 - 3xh - h^2)}{\cancel{h}} = -3x^2, \text{ i.e.,}$$

$$f'(x) = -3x^2 \longrightarrow f'(2) = -3(2)^2 = -12$$

24.) a.) $f(x) = \frac{1}{x} \rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{\frac{h}{1}} = \lim_{h \to 0} \frac{x - (x+h)}{(x+h)x} \cdot \frac{1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x} - \cancel{x} - h}{(x+h)x \cdot h} = \lim_{h \to 0} \frac{-\cancel{h}}{(x+h)x \cdot \cancel{h}} = \frac{-1}{(x)x},$$

i.e., $f'(x) = \frac{-1}{x^2} \rightarrow f'(2) = \frac{-1}{4}$

25.) $f(x) = \sqrt{x} \longrightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$$

$$= \lim_{h \to 0} \frac{(\cancel{x} + h) - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}, \text{ i.e., } f'(x) = \frac{1}{2\sqrt{x}}$$

26.) $f(x) = \frac{1}{x+1} \longrightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)+1} - \frac{1}{x+1}}{\frac{h}{1}} = \lim_{h \to 0} \frac{x+1 - (x+h+1)}{(x+h+1)(x+1)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x}+\cancel{1}-\cancel{x}-h-\cancel{1}}{(x+h+1)(x+1)\cdot h} = \lim_{h\to 0} \frac{-\cancel{h}}{(x+h+1)(x+1)\cancel{h}}$$

$$= \frac{-1}{(x+1)(x+1)} = \frac{-1}{(x+1)^2}, \text{ i.e., } f'(x) = \frac{-1}{(x+1)^2}$$

30.) $f(x) = x^2 - 3x + 1 \longrightarrow f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{h\to 0} \frac{[(x+h)^2 - 3(x+h)+1] - [x^2 - 3x + 1]}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2}+2hx+h^2-\cancel{3x}-3h+\cancel{1}-\cancel{x^2}+\cancel{3x}-\cancel{1}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2x+h-3)}{\cancel{h}} = 2x-3, \text{ i.e.,}$$

$f'(x) = 2x - 3$; pt. $(2,-1)$ so slope of tangent line is $f'(2) = 2(2) - 3 = 1$ and line is $Y - (-1) = (1)(X-2) \longrightarrow$
$Y + 1 = X - 2 \longrightarrow Y = X - 3$

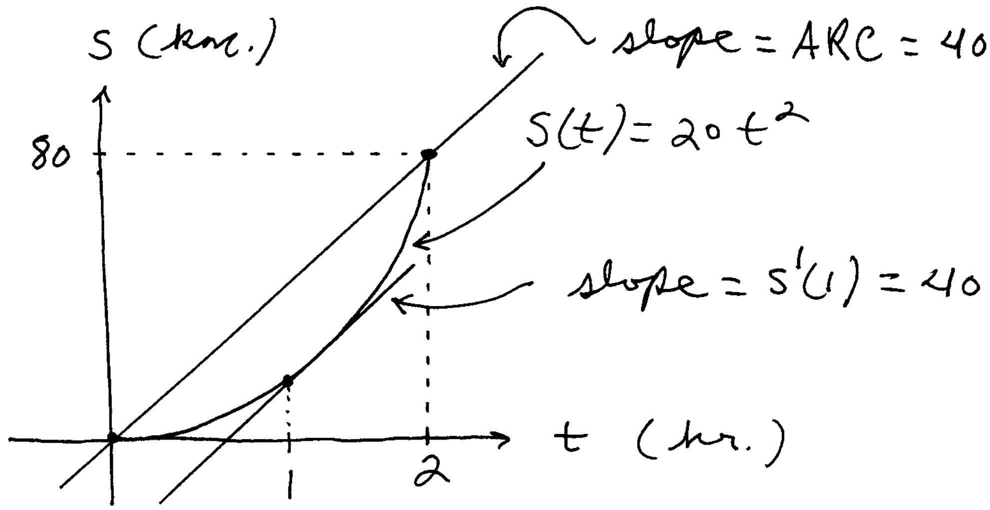
36.) If $f(x) = 4x^3$, then

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{4(a+h)^3 - 4a^3}{h}$$

38.) If $f(x) = \sin x$, then

$$f'\left(\tfrac{\pi}{6}\right) = \lim_{h\to 0} \frac{f\left(\frac{\pi}{6}+h\right)-f\left(\frac{\pi}{6}\right)}{h} = \lim_{h\to 0} \frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\frac{\pi}{6}}{h}$$

39.) a.)



b.) $\text{ARC} = \dfrac{s(2) - s(0)}{2 - 0} = \dfrac{80 - 0}{2} = 40$ km./hr.

c.) $s'(t) = \lim\limits_{h \to 0} \dfrac{s(t+h) - s(t)}{h}$

$= \lim\limits_{h \to 0} \dfrac{20(t+h)^2 - 20t^2}{h}$

$= \lim\limits_{h \to 0} \dfrac{20(t^2 + 2th + h^2) - 20t^2}{h}$

$= \lim\limits_{h \to 0} \dfrac{\cancel{20t^2} + 40th + 20h^2 - \cancel{20t^2}}{h}$

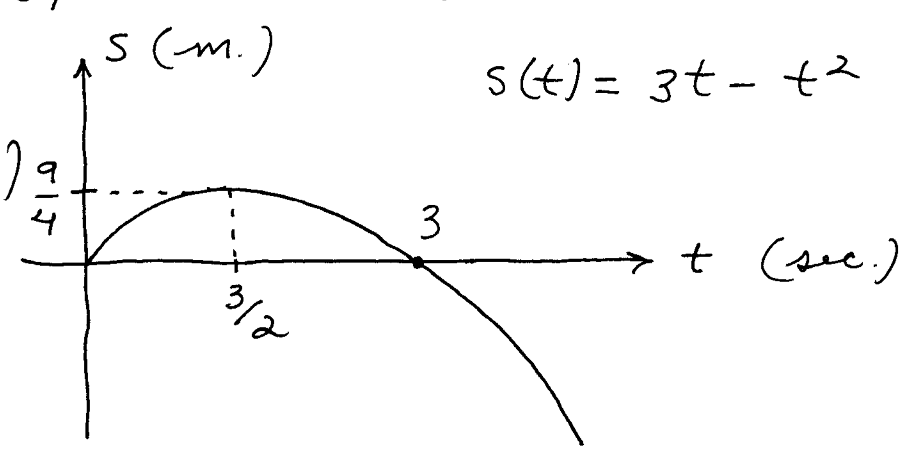
$= \lim\limits_{h \to 0} \dfrac{\cancel{h}(40t + 20h)}{\cancel{h}} = 40t$, i.e.,

$s'(t) = 40t$; then

$s'(1) = 40$ km./hr.

42.)

a.)



b.) i.) $t=0 \rightarrow s=0$

ii.) $t=3 \rightarrow s=0$

iii) $s'(t) = 3-2t = 0 \rightarrow 3 = 2t \rightarrow$

$t = \frac{3}{2}$ sec. (object changes direction) $\rightarrow$

$s\left(\frac{3}{2}\right) = 3\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2 = \frac{9}{2} - \frac{9}{4} = \frac{9}{4}$ so

particle goes $\frac{9}{4}$ m. right of $s=0$.

iv.) The particle goes infinitely far ~~to~~ the left.

v.) $s'(t) > 0$ for $0 < t < 3/2$ sec.;
$s'(t) = 0$ for $t = \frac{3}{2}$ sec.,
$s'(t) < 0$ for $t > 3/2$ sec.

vi.) velocity of particle is
$s'(t) = 3-2t$ m./sec.

49.) $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) = 0 \rightarrow$

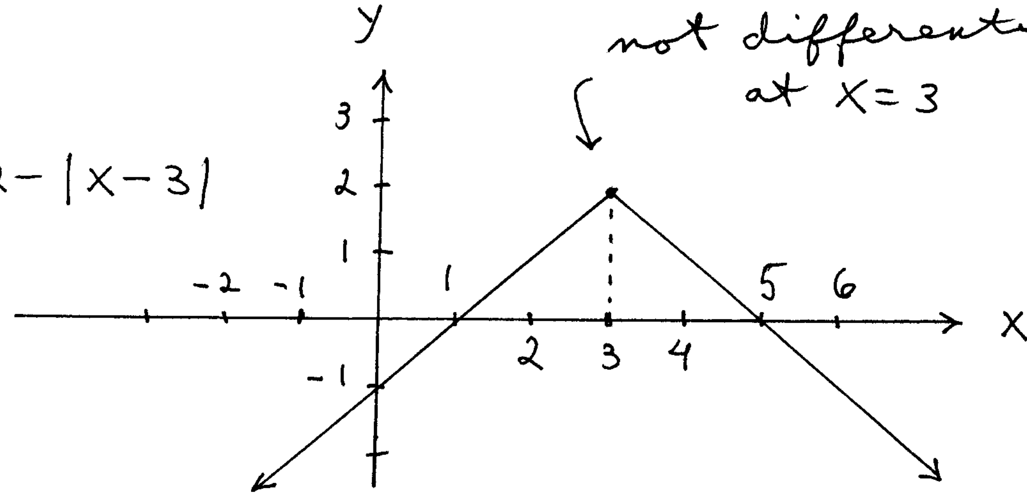
$N=0$ (No!) or $1 - \frac{N}{K} = 0 \rightarrow N = K$;

$\frac{dN}{dt} = 0$ means there is zero growth so maximum capacity has been reached.

51.) a.) False b.) True

58.)

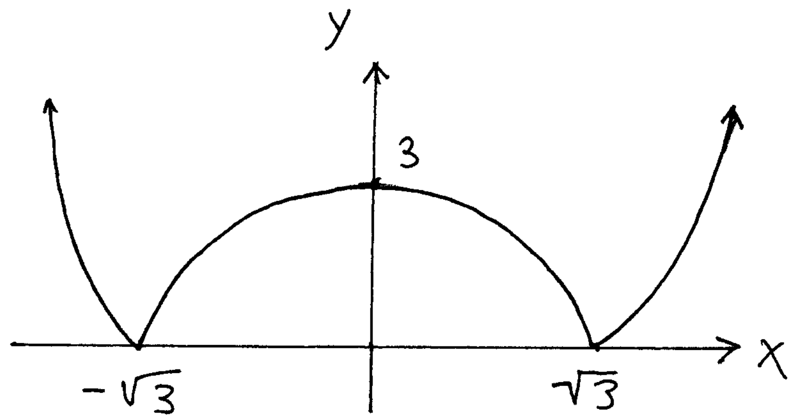
$y = 2 - |x-3|$



not differentiable at $x = 3$

64.) $y = |x^2 - 3|$

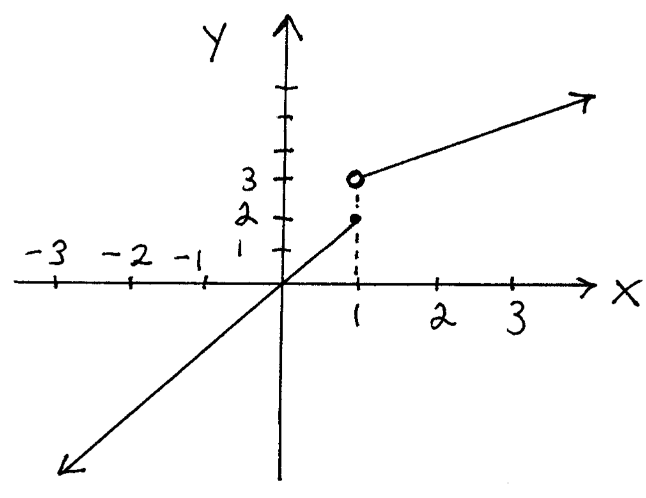
not differentiable at $x = \sqrt{3}$, $x = -\sqrt{3}$



67.)

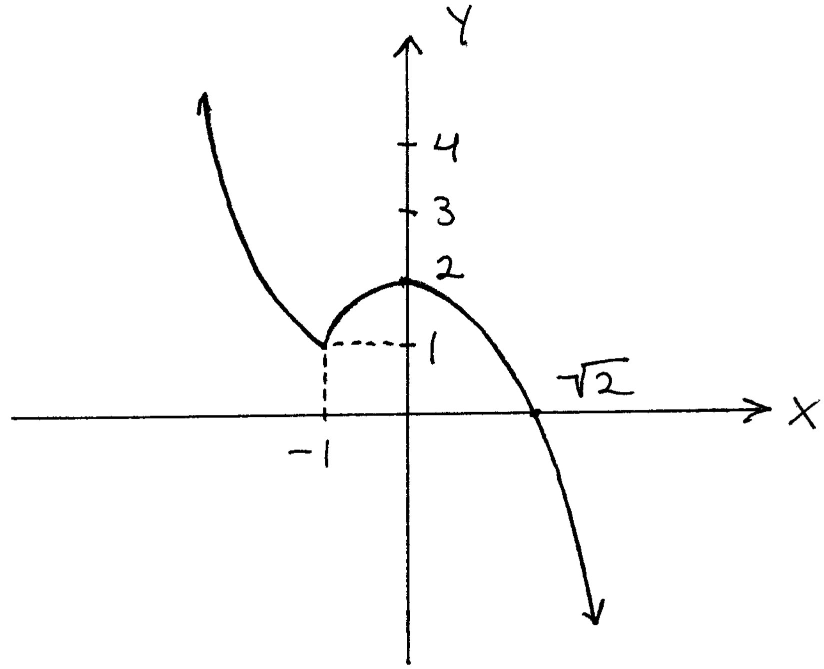
$$f(x) = \begin{cases} 2x & , \text{ for } x \le 1 \\ x+2 & , \text{ for } x > 1 \end{cases}$$

not differentiable at $x = 1$



68.)

$$f(x) = \begin{cases} x^2, & \text{for } x \le -1 \\ 2-x^2, & \text{for } x > -1 \end{cases}$$



not differentiable at $x = -1$

Worksheet 2

Sketch f' from the graph of f.