Section 4.4

2.) $f(x) = (4x+5)^3 \xrightarrow{D} f'(x) = 3(4x+5)^2 \cdot 4$

5.) $f(x) = (x^2+3)^{1/2} \xrightarrow{D} f'(x) = \frac{1}{2}(x^2+3)^{-1/2} \cdot 2x$

9.) $f(x) = \dfrac{1}{(x^3-2)^4} = (x^3-2)^{-4} \xrightarrow{D}$

$f'(x) = -4(x^3-2)^{-5} \cdot 3x^2$

12.) $f(x) = \dfrac{(1-2x^2)^3}{(3-x^2)^2} \xrightarrow{D}$

$$f'(x) = \frac{(3-x^2)^2 \cdot 3(1-2x^2)^2 \cdot (-4x) - (1-2x^2)^3 \cdot 2(3-x^2) \cdot (-2x)}{(3-x^2)^4}$$

16.) $g(t) = (t^2 + (t+1)^{1/2})^{1/2} \xrightarrow{D}$

$g'(t) = \frac{1}{2}(t^2+(t+1)^{1/2})^{-1/2} \cdot \{2t + \frac{1}{2}(t+1)^{-1/2} \cdot (1)\}$

17.) $g(t) = \left(\dfrac{t}{t-3}\right)^3 \xrightarrow{D}$

$g'(t) = 3\left(\dfrac{t}{t-3}\right)^2 \cdot \dfrac{(t-3)(1) - t(1)}{(t-3)^2}$

19.) $f(r) = (r^2-r)^3 \cdot (r+3r^3)^{-4} \xrightarrow{D}$

$f'(r) = (r^2-r)^3 \cdot -4(r+3r^3)^{-5} \cdot (1+9r^2)$

$+ 3(r^2-r)^2 \cdot (2r-1) \cdot (r+3r^3)^{-4}$

24.) $f(x) = (2-4x^2)^{1/4} \xrightarrow{D}$

$f'(x) = \frac{1}{4}(2-4x^2)^{-3/4} \cdot (-8x)$

27.) $h(t) = \left(3t + \frac{3}{t}\right)^{2/5} \xrightarrow{D}$

$$h'(t) = \frac{2}{5}\left(3t + \frac{3}{t}\right)^{-3/5} \cdot \left\{ 3 + \frac{t\cdot(0) - 3\cdot(1)}{t^2} \right\}$$

34.) a.) $f'(x) = 2x+1 \longrightarrow$

$$D\, f(x^2) = f'(x^2)\cdot D(x^2)$$

$$= \left(2(x^2)+1\right)\cdot 2x \quad , \quad \text{let } x = -1 \longrightarrow$$

$$D\, f(x^2) = \left(2(-1)^2+1\right)(-2) = -6$$

35.) b.) $f'(x) = \frac{1}{x} \longrightarrow$

$$D\, f(\sqrt{x-1}) = f'(\sqrt{x-1})\cdot D(\sqrt{x-1})$$

$$= \frac{1}{\sqrt{x-1}} \cdot \frac{1}{2}(x-1)^{-1/2}\cdot(1) = \frac{1}{2\sqrt{x-1}\,\sqrt{x-1}} = \frac{1}{2(x-1)}$$

37.) $$D\left(\frac{f(x)}{g(x)}+1\right)^2 = 2\left(\frac{f(x)}{g(x)}+1\right)\cdot \frac{g(x)\cdot f'(x) - f(x)\cdot g'(x)}{(g(x))^2}$$

38.) $$D\, f\left(\frac{1}{g(x)}\right) = f'\left(\frac{1}{g(x)}\right)\cdot \frac{g(x)\cdot(0) - (1)\cdot g'(x)}{(g(x))^2}$$

$$= f'\left(\frac{1}{g(x)}\right)\cdot \frac{-g'(x)}{(g(x))^2}$$

43.) $Y = \left(1+(3x^2-1)^3\right)^2 \xrightarrow{D}$

$$Y' = 2\left(1+(3x^2-1)^3\right)\cdot\left\{0 + 3(3x^2-1)^2\cdot(6x)\right\}$$

47.) $x^2 + Y^2 = 4 \xrightarrow{D} 2x + 2YY' = 0 \longrightarrow$

$$2YY' = -2X \rightarrow Y' = \frac{-2X}{2Y} \rightarrow Y' = \frac{-X}{Y}$$

48.) $Y = X^2 + XY \xrightarrow{D}$

$$Y' = 2X + XY' + (1)Y \rightarrow Y' - XY' = 2X + Y \rightarrow$$

$$(1-X)Y' = 2X + Y \rightarrow Y' = \frac{2X+Y}{1-X}$$

50.) $XY - Y^3 = 1 \xrightarrow{D} XY' + (1)Y - 3Y^2 \cdot Y' = 0 \rightarrow$

$$XY' - 3Y^2Y' = -Y \rightarrow (X - 3Y^2)Y' = -Y \rightarrow$$

$$Y' = \frac{-Y}{X - 3Y^2}$$

51.) $x^{1/2} y^{1/2} = x^2 + 1 \xrightarrow{D}$

$$x^{1/2} \cdot \frac{1}{2} y^{-1/2} \cdot y' + \frac{1}{2} x^{-1/2} \cdot y^{1/2} = 2x \rightarrow$$

$$\frac{1}{2}\sqrt{x} \cdot \frac{1}{\sqrt{y}} y' = 2x - \frac{1}{2} \frac{1}{\sqrt{x}} \sqrt{y} \rightarrow$$

$$y' = \frac{2x - \frac{1}{2}\sqrt{y/x}}{\frac{1}{2}\sqrt{\frac{x}{y}}}$$

52.) $\frac{1}{2xy} - y^3 = 4 \rightarrow$ (mult. by $2xy$) $\rightarrow$

$$1 - 2xy^4 = 8xy \xrightarrow{D}$$

$$0 - (2x \cdot 4y^3 \cdot y' + 2 \cdot y^4) = 8x \cdot y' + 8 \cdot y \rightarrow$$

$$-8xy^3y' - 2y^4 = 8xy' + 8y \rightarrow \text{(mult by } \tfrac{1}{2}\text{)}$$

$$-4xy^3y' - y^4 = 4xy' + 4y \rightarrow$$

$$-y^4 - 4y = 4xy^3y' + 4xy' \rightarrow$$

$$y'(4xy^3 + 4x) = -y^4 - 4y \rightarrow$$

$$y' = \frac{-y^4 - 4y}{4xy^3 + 4x}$$

53.) $\frac{x}{y} = \frac{y}{x} \rightarrow x^2 = y^2 \xrightarrow{D} \cancel{2}x = \cancel{2}yy' \rightarrow$

$y' = x/y$

54.) $\frac{x}{xy+1} = 2xy \rightarrow x = 2xy(xy+1) \rightarrow$

$x = 2x^2 \cdot y^2 + 2x \cdot y \xrightarrow{D}$

$1 = 2x^2 \cdot 2yy' + 4x \cdot y^2 + 2x \cdot y' + 2y \rightarrow$

$1 - 4xy^2 - 2y = (4x^2y + 2x)y' \rightarrow$

$$y' = \frac{1 - 4xy^2 - 2y}{4x^2y + 2x}$$

55.) $x^2 + y^2 = 25 \xrightarrow{D} 2x + 2yy' = 0 \rightarrow$

$2yy' = -2x \rightarrow y' = \frac{-2x}{2y} \rightarrow y' = \frac{-x}{y}$;

at pt. $(4,-3) \rightarrow y' = \frac{-(4)}{-3} = 4/3$;

a.) $m = y' = \frac{4}{3}$ so tangent line is

$y - (-3) = \frac{4}{3}(x-4) \rightarrow y + 3 = \frac{4}{3}x - \frac{16}{3} \rightarrow$

$y = \frac{4}{3}x - \frac{25}{3}$

60.) a.) $Y^2 = 10X^4 - X^2 \xrightarrow{D}$

$2YY' = 40X^3 - 2X \rightarrow Y' = \dfrac{40X^3 - 2X}{2Y} \rightarrow$

$Y' = \dfrac{\cancel{2}(20X^3 - X)}{\cancel{2}Y} \rightarrow Y' = \dfrac{20X^3 - X}{Y}$;

at pt. $(1, 3) \rightarrow Y' = \dfrac{20(1)^3 - 1}{3} = \dfrac{19}{3}$

61.) $X^2 + Y^2 = 1$, $\dfrac{dX}{dt} = 2$, $X = 1/2 \rightarrow$

$\left(\frac{1}{2}\right)^2 + Y^2 = 1 \rightarrow Y^2 = \frac{3}{4} \rightarrow Y = \frac{\pm\sqrt{3}}{2}$

$(Y > 0)$ so $Y = \dfrac{+\sqrt{3}}{2}$; then $\xrightarrow{D}$

$\cancel{2}X \cdot \dfrac{dX}{dt} + \cancel{2}Y \dfrac{dY}{dt} = 0 \rightarrow \left(\frac{1}{2}\right)(2) + \left(\frac{\sqrt{3}}{2}\right) \cdot \dfrac{dY}{dt} = 0$

$\rightarrow \dfrac{\sqrt{3}}{2} \cdot \dfrac{dY}{dt} = -1 \rightarrow \dfrac{dY}{dt} = \dfrac{-2}{\sqrt{3}}$.

63.) $X^2 Y = 1$, $\dfrac{dX}{dt} = 3$, $X = 2 \rightarrow 4Y = 1 \rightarrow$

$Y = 1/4$; then $\xrightarrow{D} X^2 \cdot \dfrac{dY}{dt} + 2X \cdot \dfrac{dX}{dt} \cdot Y = 0$

$\rightarrow (2)^2 \cdot \dfrac{dY}{dt} + 2(2)(3) = 0 \rightarrow \dfrac{dY}{dt} = \dfrac{-12}{4} \rightarrow$

$\dfrac{dY}{dt} = -3$.

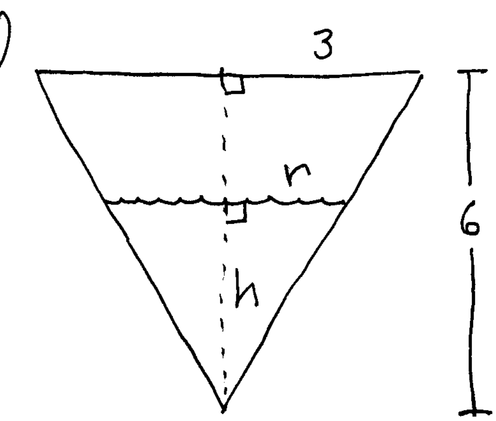
65.) $V = X^3 \xrightarrow{D} \dfrac{dV}{dt} = 3X^2 \cdot \dfrac{dX}{dt}$

69.) $V = 25\pi h$ and $\frac{dV}{dt} = \frac{250\,\ell.}{\text{min.}} \cdot \frac{1\text{ m.}^3}{1000\,\ell.} \rightarrow$

$\frac{dV}{dt} = \frac{1}{4}\ \frac{\text{m.}^3}{\text{min.}}$; $\xrightarrow{D}$ $\frac{dV}{dt} = 25\pi \cdot \frac{dh}{dt} \rightarrow$

$\frac{1}{4} = 25\pi \cdot \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{1}{100\pi}$ m./min.

70.)



By similar Δ's

$\frac{r}{h} = \frac{3}{6} \rightarrow \boxed{r = \frac{1}{2}h}$;

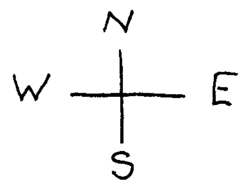
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h \rightarrow$

$\boxed{V = \frac{1}{12}\pi h^3}$; assume

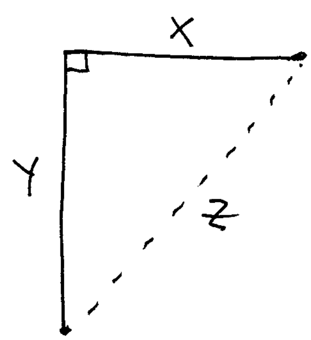
$\frac{dV}{dt} = 5$ ft.^{3}/min., find $\frac{dh}{dt}$ when $h = 2$ ft.:

$V = \frac{1}{12}\pi h^3 \xrightarrow{D} \frac{dV}{dt} = \frac{1}{12}\pi \cdot 3h^2 \cdot \frac{dh}{dt} \rightarrow$

$5 = \frac{\pi}{\cancel{4}}(\cancel{2})^{\cancel{2}} \cdot \frac{dh}{dt} \rightarrow \frac{dh}{dt} = \frac{5}{\pi}$ ft./min.



71.)



Assume $\frac{dX}{dt} = 15$ mph, $\frac{dY}{dt} = 18$ mph ; find $\frac{dZ}{dt}$ when $t = \frac{1}{3}$ hr. , $t = \frac{2}{3}$ hr. :

$X^2 + Y^2 = Z^2 \xrightarrow{D}$

$$2X \cdot \frac{dX}{dt} + 2Y \cdot \frac{dY}{dt} = 2Z \frac{dZ}{dt} \rightarrow X \cdot \frac{dX}{dt} + Y \cdot \frac{dY}{dt} = Z \cdot \frac{dZ}{dt}$$

$$\rightarrow \boxed{\frac{dZ}{dt} = \frac{1}{Z}\left(X \cdot \frac{dX}{dt} + Y \cdot \frac{dY}{dt}\right)} \quad (*) \quad ;$$

a.) If $t = \frac{1}{3}$ hr. $\rightarrow$ $X = (15)(\frac{1}{3}) = 5$ mi. , $Y = (18)(\frac{1}{3}) = 6$ mi. , and $(5)^2 + (6)^2 = Z^2 \rightarrow$ $Z^2 = 61 \rightarrow Z = \sqrt{61} \approx 7.8$ mi. $\rightarrow$ (plug in (*))

$$\rightarrow \frac{dZ}{dt} = \frac{1}{\sqrt{61}}((5)\cdot(15) + (6)(18)) \approx 23.4 \text{ mph}$$

b.) If $t = \frac{2}{3}$ hr. $\rightarrow$ $X = (15)(\frac{2}{3}) = 10$ mi. , $Y = (18)(\frac{2}{3}) = 12$ mi. , and $(10)^2 + (12)^2 = Z^2 \rightarrow$ $Z^2 = 244 \rightarrow Z = \sqrt{244} \approx 15.6$ mi. $\rightarrow$ (plug in (*))

$$\rightarrow \frac{dZ}{dt} = \frac{1}{\sqrt{244}}((10)(15) + (12)(18)) \approx 23.4 \text{ mph}$$

74.) $f(x) = (x^2 - 3)^2 \xrightarrow{D}$

$f'(x) = 2(x^2 - 3) \cdot 2x = 4x^3 - 12x \xrightarrow{D}$

$f''(x) = 12x^2 - 12$

80.) $f(x) = \frac{x}{x+1} \xrightarrow{D}$

$$f'(x) = \frac{(x+1)(1) - x\cdot(1)}{(x+1)^2} = \frac{1}{(x+1)^2} = (x+1)^{-2} \xrightarrow{D}$$

$$f''(x) = -2(x+1)^{-3}\cdot(1)$$

86.) a.) $s(t) = t^2 - 3t \xrightarrow{D}$

vel. $s'(t) = 2t - 3 \rightarrow s'(1) = 2 - 3 = -1$; $\xrightarrow{D}$

acc. $s''(t) = 2 \rightarrow s''(1) = 2$

b.) $s(t) = \sqrt{t^2+1} \xrightarrow{D}$

vel. $s'(t) = \frac{1}{2}(t^2+1)^{-1/2}\cdot(2t) = \frac{t}{\sqrt{t^2+1}} \rightarrow$

$s'(1) = 1/\sqrt{2}$; $\xrightarrow{D}$

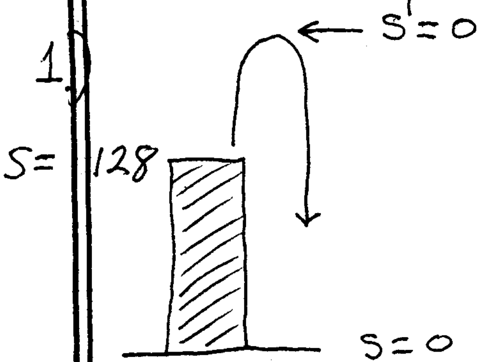
acc. $$s''(t) = \frac{\sqrt{t^2+1}\cdot(1) - t\cdot\frac{1}{2}(t^2+1)^{-1/2}\cdot 2t}{(\sqrt{t^2+1})^2}$$

$$= \frac{\frac{\sqrt{t^2+1}}{1} \times \frac{t^2}{\sqrt{t^2+1}}}{\frac{t^2+1}{1}} = \frac{\cancel{t^2}+1-\cancel{t^2}}{(t^2+1)^{1/2}} \cdot \frac{1}{(t^2+1)^1} \rightarrow$$

$$s''(t) = \frac{1}{(t^2+1)^{3/2}} \rightarrow s''(1) = \frac{1}{2^{3/2}}$$

Gravity Problems

1.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + 112t + 128}$

$\xrightarrow{D} \boxed{s'(t) = -32t + 112}$

a.) highest point : $s'(t) = 0$

$\rightarrow -32t + 112 = 0 \rightarrow t = \frac{112}{32} = \underline{3.5 \text{ sec.}} \rightarrow$

$s(3.5) = -16(3.5)^2 + 112(3.5) + 128 = 324 \text{ ft.}$

b.) hit ground : $s(t) = 0 \rightarrow$

$-16t^2 + 112t + 128 = 0 \rightarrow -16(t^2 - 7t - 8) = 0 \rightarrow$

$-16(t-8)(t+1) = 0 \rightarrow t = -1$ (NO) or

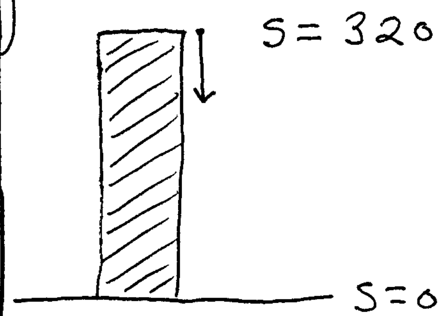
$t = 8$ sec.

c.) i.) $s'(3) = -32(3) + 112 = 16$ ft./sec

ii.) $s'(4) = -32(4) + 112 = -16$ ft./sec.

iii.) $s'(8) = -32(8) + 112 = -144$ ft./sec.

2.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 - 16t + 320} \rightarrow$

$\boxed{s'(t) = -32t - 16}$

a.) hit ground : $s(t) = 0 \rightarrow$

$-16t^2 - 16t + 320 = 0 \rightarrow -16(t^2 + t - 20) = 0 \rightarrow$

$-16(t-4)(t+5) = 0 \rightarrow t = -5$ (NO) or

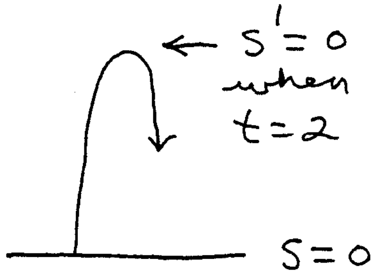
$t = 4$ sec.

b.) i.) $s'(1) = -32(1) - 16 = -48$ ft./sec.

ii.) $s'(2) = -32(2) - 16 = -80$ ft./sec.

iii.) $s'(4) = -32(4) - 16 = -144$ ft./sec.

3.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + v_0 t} \rightarrow$

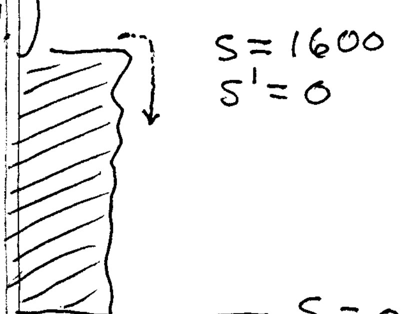
$\boxed{s'(t) = -32t + v_0}$

and $s'(2) = 0 \rightarrow -32(2) + v_0 = 0$

$\rightarrow$ b.) $v_0 = 64$ ft./sec.

a.) $s(2) = -16(2)^2 + 64(2) = 64$ ft.

4.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + 1600} \rightarrow$

$\boxed{s'(t) = -32t}$

a.) hit ground: $s(t) = 0 \rightarrow$

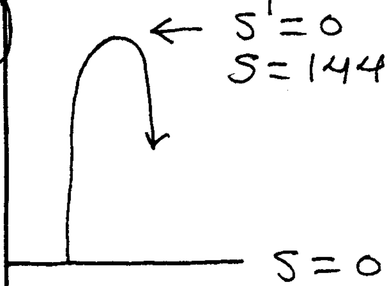
$-16t^2 + 1600 = 0 \rightarrow 16t^2 = 1600 \rightarrow t^2 = 100 \rightarrow$

$t = 10$ sec.

b.) i.) $s'(5) = -32(5) = -160$ ft./sec.

ii.) $s'(10) = -32(10) = -320$ ft./sec;

$$-\frac{320 \text{ ft.}}{\text{sec}} \times \cdot \frac{1 \text{ mi}}{5280 \text{ ft.}} \cdot \frac{3600 \text{ sec.}}{1 \text{ hr.}} \approx -218.2 \text{ mph}$$

5.) 

$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + v_0 t} \rightarrow$

$\boxed{s'(t) = -32t} + v_0;$

Let $t = T$ be time required to reach highest point. Then

$$\left.\begin{array}{l} s'(T) = -32T + v_0 = 0 \\ s(T) = -16T^2 + v_0 T = 144 \end{array}\right\} \rightarrow$$

$\boxed{v_0 = 32T} \xrightarrow{(SUB)} -16T^2 + (32T)T = 144 \rightarrow$

$16T^2 = 144 \rightarrow T^2 = 9 \rightarrow T = 3$ sec.

a.) $T = 3$ sec

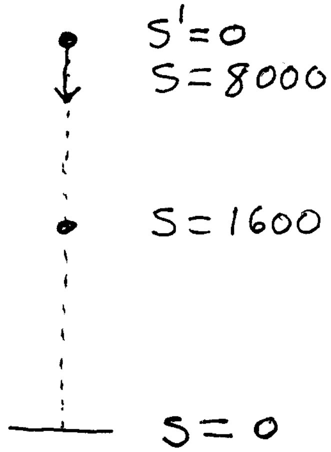
c.) $v_0 = 32T = 32(3) = 96$ ft./sec.

b.) hit ground : $s(t) = 0 \rightarrow$

$-16t^2 + 96t = 0 \rightarrow 16t(6 - t) = 0 \rightarrow$

$t = 0$ and $t = 6$ sec.

d.) You know.

6.) 

$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$

$\boxed{s(t) = -16t^2 + 8000} \rightarrow$

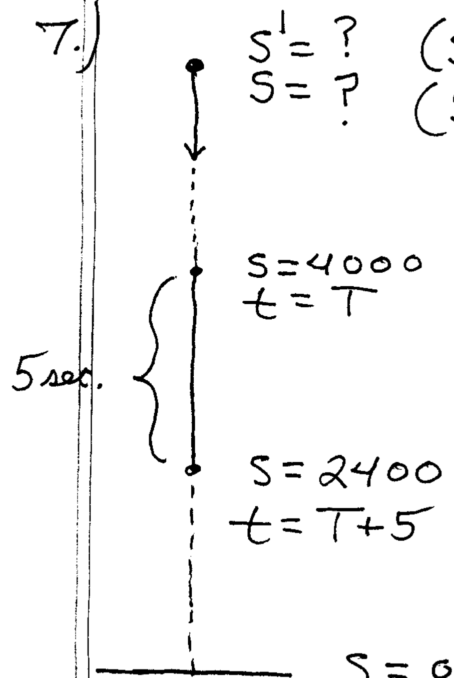
$\boxed{s'(t) = -32t}$

a.) $s(t) = 1600 \rightarrow -16t^2 + 8000 = 1600$

$\rightarrow 16t^2 = 6400 \rightarrow t^2 = 400 \rightarrow t = 20 \text{ sec.}$

b.) $s'(20) = -32(20) = -640$ ft./sec.

7.)



$s(t) = -16t^2 + v_0 t + s_0 \rightarrow$
$s'(t) = -32t + v_0$;

a.) $s'(10) = -400 \rightarrow$
$-32(10) + v_0 = -400 \rightarrow$
$v_0 = -80$ ft./sec.

b.) $\left.\begin{array}{l} s(T) = 4000 \\ s(T+5) = 2400 \end{array}\right\} \rightarrow$

$\left.\begin{array}{r} -16T^2 - 80T + s_0 = 4000 \\ -16(T+5)^2 - 80(T+5) + s_0 = 2400 \end{array}\right\} \rightarrow$

$\boxed{s_0 = 4000 + 80T + 16T^2} \rightarrow$ (SUB) $\rightarrow$

$-16(T^2 + 10T + 25) - 80T - 400$
$+ (4000 + 80T + 16T^2) = 2400 \rightarrow$

$\cancel{-16T^2} - 160T - 400 - \cancel{80T} - 400$
$+ 4000 + \cancel{80T} + \cancel{16T^2} = 2400 \rightarrow$

$160T = 800 \rightarrow T = 5 \text{ sec.} \rightarrow$

$s_0 = 4000 + 80(5) + 16(5)^2 \rightarrow$

$s_0 = 4800$ ft.

c.) $s(t) = -16t^2 - 80t + 4800$,

strike ground : $s(t) = 0 \rightarrow$

$-16t^2 - 80t + 4800 = 0 \rightarrow$

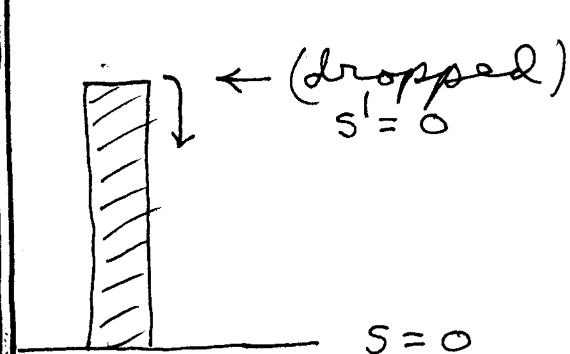
$-16(t^2 + 5t - 300) = 0 \rightarrow$

$-16(t - 15)(t + 20) = 0 \rightarrow$

$t = 15$ sec. $\quad t \neq -20$

d.) Snapple Peach Ice Tea

8.)



Let H be height of building. Then

$s(t) = -16t^2 + (0)t + H \rightarrow$

$\boxed{s(t) = -16t^2 + H} \xrightarrow{D}$

$\boxed{s'(t) = -32t}$;

a.) Given $s(5) = 0$ ft. $\rightarrow -16(5)^2 + H = 0$

$\rightarrow \boxed{H = 400 \text{ ft}}$

b.) $s'(1) = -32(1) = -32$ ft./sec. ;

$s'(3) = -32(3) = -96$ ft./sec.

c.) $s'(5) = -32(5) = -160$ ft./sec.

$$= \frac{-160 \text{ ft.}}{\text{sec.}} \times \frac{1 \text{ mi.}}{5280 \text{ ft.}} \times \frac{3600 \text{ sec.}}{1 \text{ hr.}} \approx 109.1 \text{ mph}$$