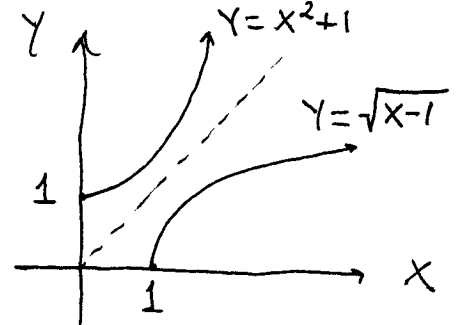


Section 4.7

2.) $f(x) = \sqrt{x-1} \rightarrow y = \sqrt{x-1} \rightarrow (\text{switch}) \rightarrow$

$x = \sqrt{y-1} \rightarrow x^2 = y-1 \rightarrow y = x^2 + 1 \rightarrow$

$f^{-1}(x) = x^2 + 1 :$



i.) $D f^{-1}(x) = D(x^2 + 1) = 2x$

ii.) $D f^{-1}(x) = \frac{1}{f'(f^{-1}(x))}$

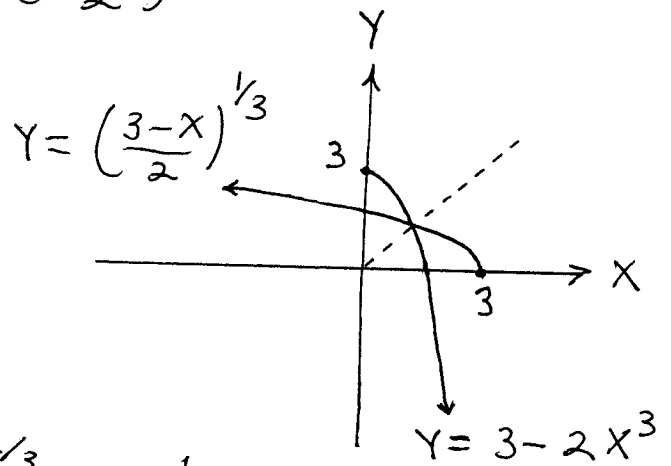
$\left\{ f'(x) = \frac{1}{2}(x-1)^{-1/2} = \frac{1}{2\sqrt{x-1}} \right\}$

$= \frac{1}{\frac{1}{2\sqrt{(x^2+1)-1}}} = \frac{1}{\frac{1}{2\sqrt{x^2}}} = \frac{1}{\frac{1}{2|x|}} = 2|x| = 2x$
since $x \geq 0$

5.) $f(x) = 3 - 2x^3$ for $x \geq 0 \rightarrow y = 3 - 2x^3 \rightarrow$
(switch) $\rightarrow x = 3 - 2y^3 \rightarrow 2y^3 = 3 - x \rightarrow$

$y^3 = \frac{3-x}{2} \rightarrow y = \left(\frac{3-x}{2}\right)^{1/3} \rightarrow$

$f^{-1}(x) = \left(\frac{3-x}{2}\right)^{1/3} :$



i.) $D f^{-1}(x) = D \left(\frac{3-x}{2}\right)^{1/3}$

$= \frac{1}{3} \left(\frac{3-x}{2}\right)^{-2/3} \cdot \left(-\frac{1}{2}\right)$

$= \frac{-1}{6} \cdot \frac{1}{\frac{(3-x)^{2/3}}{2^{2/3}}} = \frac{-2^{2/3}}{3 \cdot 2} \cdot \frac{1}{(3-x)^{2/3}}$

$= \frac{-1}{3 \cdot 2^{1/3}} \cdot \frac{1}{(3-x)^{2/3}}$

$$\begin{aligned}
 \text{ii.) } D f^{-1}(x) &= \frac{1}{f'(f^{-1}(x))} \quad \{f'(x) = -6x^2\} \\
 &= \frac{1}{-6 \left[\left(\frac{3-x}{2} \right)^{1/3} \right]^2} = \frac{-1}{6 \cdot \frac{(3-x)^{2/3}}{2^{2/3}}} = \frac{-2^{2/3}}{3 \cdot 2 \cdot (3-x)^{2/3}} \\
 &= \frac{-1}{3 \cdot 2^{1/3} (3-x)^{2/3}}
 \end{aligned}$$

$$\begin{aligned}
 7.) \quad f(x) &= 2x^2 - 2, \quad f(1) = 0 \rightarrow f^{-1}(0) = 1, \text{ so} \\
 D f^{-1}(x) &= \frac{1}{f'(f^{-1}(x))} \quad \{f'(x) = 4x\} \\
 &= \frac{1}{4 f^{-1}(x)} \quad \text{so } D f^{-1}(0) = \frac{1}{4 f^{-1}(0)} = \frac{1}{4 \cdot 1} = \frac{1}{4}
 \end{aligned}$$

$$\begin{aligned}
 11.) \quad f(x) &= x + e^x, \quad f'(x) = 1 + e^x, \quad f(0) = 1 \rightarrow \\
 &f^{-1}(1) = 0, \text{ so} \\
 D f^{-1}(x) &= \frac{1}{f'(f^{-1}(x))} = \frac{1}{1 + e^{f^{-1}(x)}}, \text{ so} \\
 D f^{-1}(1) &= \frac{1}{1 + e^{f^{-1}(1)}} = \frac{1}{1 + e^0} = \frac{1}{1+1} = \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 15.) \quad f(x) &= x^2 + \tan x, \quad f'(x) = 2x + \sec^2 x, \\
 &f(0) = 0 \rightarrow f^{-1}(0) = 0, \text{ so} \\
 D f^{-1}(x) &= \frac{1}{f'(f^{-1}(x))} = \frac{1}{2f^{-1}(x) + \sec^2(f^{-1}(x))} \rightarrow \\
 D f^{-1}(0) &= \frac{1}{2f^{-1}(0) + \sec^2(f^{-1}(0))} = \frac{1}{2(0) + \sec^2 0}
 \end{aligned}$$

$$= \frac{1}{0+(1)^2} = 1$$

$$26.) f(x) = \ln(4-3x) \xrightarrow{D} f'(x) = \frac{1}{4-3x} \cdot (-3)$$

$$31.) f(x) = (\ln x)^2 \xrightarrow{D} f'(x) = 2 \ln x \cdot \frac{1}{x}$$

$$34.) f(x) = (\ln(1-x^2))^3 \xrightarrow{D}$$

$$f'(x) = 3(\ln(1-x^2))^2 \cdot \frac{1}{1-x^2} \cdot -2x$$

$$36.) f(x) = \ln(2x^2-x)^{1/2} \xrightarrow{D}$$

$$f'(x) = \frac{1}{(2x^2-x)^{1/2}} \cdot \frac{1}{2}(2x^2-x)^{-1/2} \cdot (4x-1)$$

$$37.) f(x) = \ln\left(\frac{x}{x+1}\right) \xrightarrow{D}$$

$$f'(x) = \frac{1}{\frac{x}{x+1}} \cdot \frac{(x+1)(1) - x(1)}{(x+1)^2}$$

$$45.) f(x) = \ln(\tan(x^2)) \xrightarrow{D}$$

$$f'(x) = \frac{1}{\tan(x^2)} \cdot \sec^2(x^2) \cdot 2x$$

$$46.) g(s) = \ln(\sin^2(3s)) \xrightarrow{D}$$

$$g'(s) = \frac{1}{\sin^2(3s)} \cdot 2 \sin(3s) \cdot \cos(3s) \cdot 3$$

$$47.) f(x) = x \ln x \xrightarrow{D} f'(x) = x \cdot \frac{1}{x} + (1) \ln x$$

$$49.) f(x) = \frac{\ln x}{x} \xrightarrow{D} f'(x) = \frac{x \cdot \frac{1}{x} - \ln x \cdot (1)}{x^2}$$

$$51.) h(t) = \sin(\ln(3t)) \xrightarrow{D}$$

$$h'(t) = \cos(\ln(3t)) \cdot \frac{1}{3t} \cdot 3$$

$$58.) f(x) = \log(\tan(x^2))^{1/3} \xrightarrow{D}$$

$$f'(x) = \frac{1}{(\tan(x^2))^{1/3}} \cdot \frac{1}{3} (\tan(x^2))^{-2/3} \cdot \sec^2(x^2) \cdot 2x \cdot \frac{1}{\ln 10}$$

$$60.) g(s) = \log_5(3^s - 2) \xrightarrow{D}$$

$$= \frac{1}{3^s - 2} \cdot 3^s \cdot \ln 3 \cdot \frac{1}{\ln 5}$$

$$63.) f(x) = 2 \cdot x^x \rightarrow \ln f(x) = \ln(2 \cdot x^x) \rightarrow$$

$$\ln f(x) = \ln 2 + \ln x^x = \ln 2 + x \cdot \ln x \xrightarrow{D}$$

$$\frac{1}{f(x)} f'(x) = 0 + x \cdot \frac{1}{x} + (1) \ln x \rightarrow f'(x) = f(x)(1 + \ln x)$$

$$\rightarrow f'(x) = 2 \cdot x^x (1 + \ln x)$$

$$67.) f(x) = x^{\ln x} \rightarrow \ln f(x) = \ln x^{\ln x} \rightarrow$$

$$\ln f(x) = \ln x \cdot \ln x = (\ln x)^2 \xrightarrow{D}$$

$$\frac{1}{f(x)} \cdot f'(x) = 2(\ln x) \cdot \frac{1}{x} \rightarrow$$

$$f'(x) = x^{\ln x} \left(\frac{2 \ln x}{x} \right) = 2 x^{\ln x - 1} \cdot \ln x$$

$$71.) y = x^{x^x} \rightarrow \ln y = \ln x^{x^x} = x^x \cdot \ln x \rightarrow$$

$$\ln(\ln y) = \ln(x^x \cdot \ln x) \rightarrow$$

$$\ln(\ln Y) = \ln x^x + \ln(\ln x) \rightarrow$$

$$\ln(\ln Y) = x \cdot \ln x + \ln(\ln x) \xrightarrow{D}$$

$$\frac{1}{\ln Y} \cdot \frac{1}{Y} \cdot Y' = x \cdot \frac{1}{x} + (1) \ln x + \frac{1}{\ln x} \cdot \frac{1}{x} \rightarrow$$

$$Y' = Y \ln Y \cdot \left[1 + \ln x + \frac{1}{x \ln x} \right]$$

$$= x^{x^x} \ln x^{x^x} \cdot \left[1 + \ln x + \frac{1}{x \ln x} \right]$$

$$= x^{x^x} \cdot x^x \cdot \ln x \cdot \left[1 + \ln x + \frac{1}{x \ln x} \right]$$

$$72.) Y = (x^x)^x = x^{x^2} \rightarrow \ln Y = \ln x^{x^2} = x^2 \ln x \xrightarrow{D}$$

$$\frac{1}{Y} Y' = x^2 \cdot \frac{1}{x} + 2x \cdot \ln x \rightarrow$$

$$Y' = x^{x^2} \cdot [x + 2x \ln x]$$

$$73.) Y = x^{\cos x} \rightarrow \ln Y = \ln x^{\cos x} = \cos x \cdot \ln x$$

$$\xrightarrow{D} \frac{1}{Y} Y' = \cos x \cdot \frac{1}{x} + -\sin x \cdot \ln x$$

$$\rightarrow Y' = x^{\cos x} \cdot \left[\frac{\cos x}{x} - \sin x \cdot \ln x \right]$$

$$76.) Y = \frac{e^{x-1} \cdot \sin^2 x}{(x^2+5)^{2x}} \rightarrow \ln Y = \ln \left(\frac{e^{x-1} \cdot \sin^2 x}{(x^2+5)^{2x}} \right)$$

$$= \ln(e^{x-1} \cdot \sin^2 x) - \ln(x^2+5)^{2x}$$

$$= \ln e^{x-1} + \ln(\sin x)^2 - 2x \cdot \ln(x^2+5)$$

$$= (x-1) + 2 \ln(\sin x) - 2x \cdot \ln(x^2+5) \xrightarrow{D}$$

$$\frac{1}{Y} Y' = 1 + 2 \cdot \frac{1}{\sin x} \cdot \cos x$$

$$- \left[2x \cdot \frac{1}{x^2+5} \cdot 2x + 2 \cdot \ln(x^2+5) \right] \rightarrow$$

$$Y' = \frac{e^{x-1} \sin^2 x}{(x^2+5)^{2x}} \cdot \left\{ 1 + \cot x - \frac{4x^2}{x^2+5} - 2 \ln(x^2+5) \right\}$$