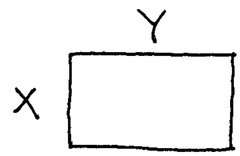
Section 5.4

1.) 

area $XY = 25 \rightarrow \boxed{Y = \frac{25}{X}}$;

minimize perimeter

$P = 2X + 2Y = 2X + 2\left(\frac{25}{X}\right) = 2X + \frac{50}{X} \rightarrow$

$\boxed{P = 2X + \frac{50}{X}} \xrightarrow{D} P' = 2 - \frac{50}{X^2} = 0 \rightarrow$

$2 = \frac{50}{X^2} \rightarrow X^2 = 25 \rightarrow X = 5$ in.

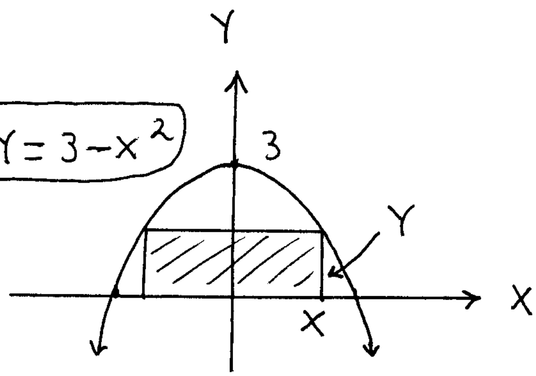
$-\ \ 0\ \ +$ P'

$X = 5$ in.

$Y = 5$ in.

min $P = 20$ in.

$\boxed{Y = 3 - x^2}$

3.) 

maximize area

$A = (2X)Y = 2XY \rightarrow$

$A = 2X \cdot (3 - X^2) = 6X - 2X^3 \rightarrow \boxed{A = 6X - 2X^3} \xrightarrow{D}$

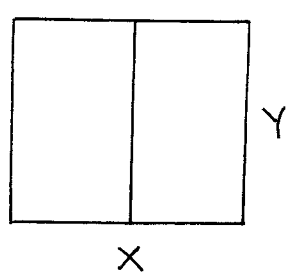
$A' = 6 - 6X^2 = 6(1 - X^2) = 0 \rightarrow X = 1$

$+\ \ 0\ \ -$ A'

$X = 1$

$Y = 2$

max. $A = 4$

4.) 

area $XY = 384 \rightarrow \boxed{Y = \frac{384}{X}}$;

minimize $L = 2X + 3Y = 2X + 3\left(\frac{384}{X}\right) \rightarrow$

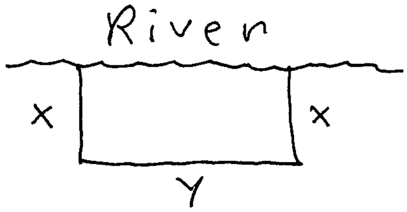
$\boxed{L = 2X + \frac{1152}{X}} \xrightarrow{D} L' = 2 - \frac{1152}{X^2} = 0 \rightarrow$

$2 = \frac{1152}{X^2} \rightarrow X^2 = 576 \rightarrow X = 24$ ft.

$-\ \ 0\ \ +$ L'

$X = 24$ ft., $Y = 16$ ft., min. $L = 96$ ft.

5.)



Length $2X+Y = 320$ ft. $\rightarrow$ $\boxed{Y = 320 - 2X}$;

maximize area $A = XY = X(320-2X) \rightarrow$

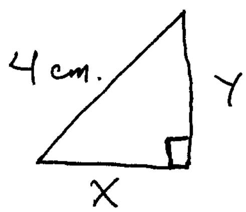
$\boxed{A = 320X - 2X^2} \xrightarrow{D} A' = 320 - 4X = 0 \rightarrow$

$X = 80$ ft.

$$\begin{array}{c} + \quad 0 \quad - \\ \hline \end{array} \; A'$$

$X = 80$ ft.
$Y = 160$ ft.
max. $A = 12{,}800$ ft.^{2}

6.)



Maximize area

$A = \dfrac{XY}{2}$ and $\boxed{X^2+Y^2 = 4^2} \rightarrow$

$Y = \sqrt{16-X^2} \longrightarrow \boxed{A = \dfrac{X}{2}\sqrt{16-X^2}} \xrightarrow{D}$

$$A' = \frac{X}{2}\cdot\frac{1}{\cancel{2}}(16-X^2)^{-1/2}\cdot(-\cancel{2}X) + \left(\frac{1}{2}\right)\sqrt{16-X^2}$$

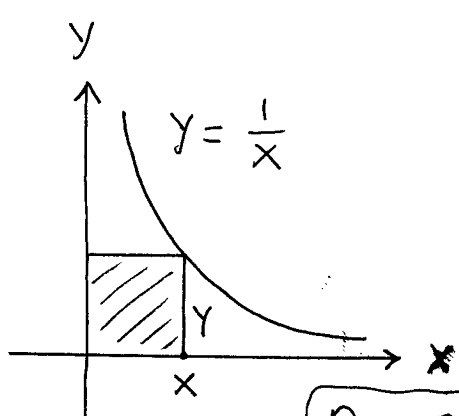
$$= \frac{-X^2}{2\sqrt{16-X^2}} + \frac{\sqrt{16-X^2}}{2} = \frac{-X^2+(16-X^2)}{2\sqrt{16-X^2}}$$

$$= \frac{16-2X^2}{2\sqrt{16-X^2}} = \frac{\cancel{2}(8-X^2)}{\cancel{2}\sqrt{16-X^2}} = 0 \longrightarrow X = \sqrt{8} \text{ cm.}$$

$$\begin{array}{c} + \quad 0 \quad - \\ \hline \end{array} \; A'$$

$X = \sqrt{8}$ cm.
$Y = \sqrt{8}$ cm.
max. $A = 4$ cm.^{2}

9.)



Minimize perimeter

$P = 2X + 2Y = 2X + 2\left(\dfrac{1}{X}\right) \rightarrow$

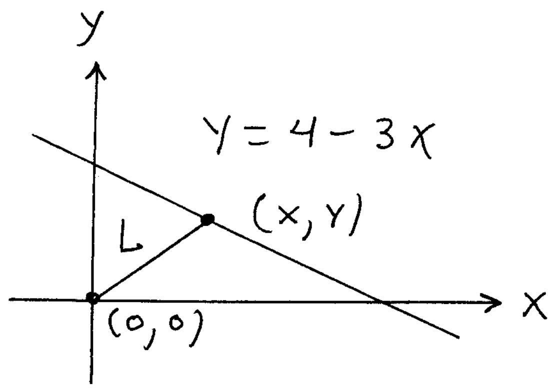
$\boxed{P = 2X + \dfrac{2}{X}} \xrightarrow{D} P' = 2 - \dfrac{2}{X^2} = 0 \rightarrow$

$$2 = \frac{2}{x^2} \rightarrow x^2 = 1 \rightarrow x = 1$$

$$\begin{array}{c} - \quad 0 \quad + \\ \hline x=1 \\ y=1 \end{array} \quad P'$$

max. $P = 4$

11.)



minimize length

$$L = \sqrt{(x-0)^2 + (y-0)^2}$$

$$\rightarrow L = \sqrt{x^2+y^2} = \sqrt{x^2+(4-3x)^2} \longrightarrow$$

$$\boxed{L = \sqrt{x^2+(4-3x)^2}} \xrightarrow{D} L' = \frac{1}{2}\left(x^2+(4-3x)^2\right)^{-1/2} \cdot \{2x + 2(4-3x)\cdot(-3)\}$$

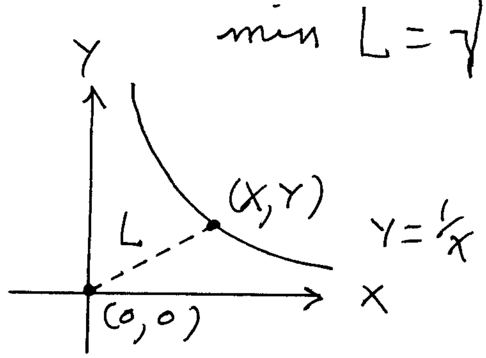
$$= \frac{2x - 24 + 18x}{2\sqrt{x^2+(4-3x)^2}} = \frac{20x-24}{2\sqrt{x^2+(4-3x)^2}} = \frac{4(5x-6)}{2\sqrt{x^2+(4-3x)^2}} = 0$$

$$\rightarrow x = 6/5$$

$$\begin{array}{c} - \quad 0 \quad + \\ \hline x=6/5 \\ y=2/5 \end{array} \quad L'$$

$$\text{min } L = \sqrt{\frac{36}{25}+\frac{4}{25}} = \sqrt{\frac{40}{25}} = \frac{2\sqrt{10}}{5}$$

13.)



minimize distance

$$L = \sqrt{(x-0)^2+(y-0)^2}$$

$$= \sqrt{x^2+y^2} = \sqrt{x^2+\left(\frac{1}{x}\right)^2}$$

$$\rightarrow L = \sqrt{x^2+\frac{1}{x^2}} \xrightarrow{D}$$

$$L' = \frac{1}{2}\left(x^2+\frac{1}{x^2}\right)^{-1/2} \cdot \left\{2x - \frac{2}{x^3}\right\} = \frac{\cancel{2}\left(x - \frac{1}{x^3}\right)}{\cancel{2}\sqrt{x^2+\frac{1}{x^2}}} = 0$$

$\rightarrow \quad x - \frac{1}{x^3} = 0 \rightarrow x = \frac{1}{x^3} \rightarrow x^4 = 1 \rightarrow x = 1$

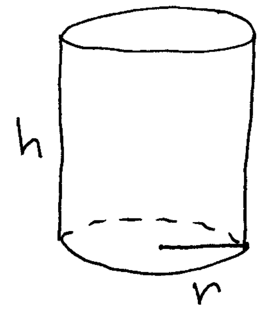
NO

$- \quad 0 \quad + \qquad L'$

$x=0 \qquad x=1$

$Y = 1$

min. $L = \sqrt{2}$

17.) 

Volume $\pi r^2 h = 1000 \text{ cm}^3$

$\rightarrow \boxed{h = \frac{1000}{\pi r^2}}$;

minimize surface area

$S = S_{top} + S_{bottom} + S_{side}$

$= \pi r^2 + \pi r^2 + 2\pi r \cdot h$

$= 2\pi r^2 + 2\cancel{\pi}\cancel{r} \cdot \left(\frac{1000}{\cancel{\pi} r^{\cancel{2}}}\right) \rightarrow$

$\boxed{S = 2\pi r^2 + \frac{2000}{r}} \xrightarrow{D} S' = 4\pi r - \frac{2000}{r^2}$

$= \frac{4\pi r^3 - 2000}{r^2} = 0 \rightarrow 4\pi r^3 - 2000 = 0 \rightarrow$

$4\pi r^3 = 2000 \rightarrow r^3 = \frac{2000}{4\pi} \rightarrow$

$r = \left(\frac{500}{\pi}\right)^{1/3} \approx 5.42$

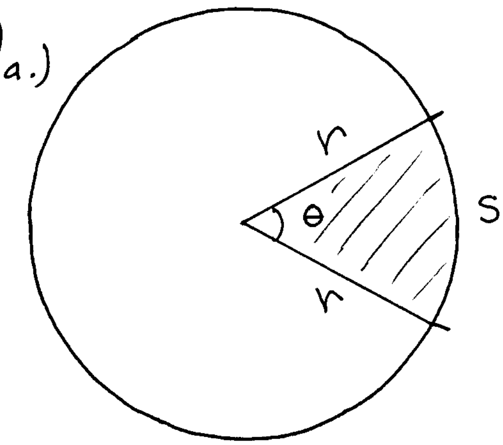
$- \quad 0 \quad + \qquad S'$

$r = \left(\frac{500}{\pi}\right)^{1/3} \approx 5.42 \text{ cm.}$

$h = \frac{1000}{\pi \left(\frac{500}{\pi}\right)^{2/3}} = \frac{500 \cdot 2}{\pi \cdot \frac{500^{2/3}}{\pi^{2/3}}} = 2\left(\frac{500}{\pi}\right)^{1/3} \approx 10.84 \text{ cm.}$

min $S = 2\pi \left(\frac{500}{\pi}\right)^{2/3} + \frac{2000}{\left(\frac{500}{\pi}\right)^{1/3}} \text{ cm.}^2$

19.) a.)



area of sector is

$$A = \frac{\theta}{2\pi} \cdot \pi r^2 = \frac{1}{2}\theta r^2$$

and $A = 2$ so

$$2 = \frac{1}{2}\theta r^2 \rightarrow \boxed{\theta = \frac{4}{r^2}} \; ;$$

minimize perimeter

$$P = r + r + s = 2r + r\theta = 2r + r \cdot \left(\frac{4}{r^2}\right) \rightarrow$$

$$\boxed{P = 2r + \frac{4}{r}} \xrightarrow{D} P' = 2 - \frac{4}{r^2} = \frac{2r^2 - 4}{r^2} = 0$$

$$\rightarrow 2r^2 - 4 = 0 \rightarrow r^2 = 2 \rightarrow r = \sqrt{2} \; ;$$

NO	$-$	0	$+$	P'
$r = 0$		$r = \sqrt{2}$		
		$\theta = 2$		

$$\text{min. } P = 2\sqrt{2} + \frac{4}{\sqrt{2}} = 2\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}$$

I.)

volume $64\pi = \pi r^2 h \rightarrow h = \frac{64}{r^2}$,

minimize surface area

$$S = \pi r^2 + 2\pi r h = \pi r^2 + 2\pi r\left(\frac{64}{r^2}\right) \rightarrow$$

$$S = \pi r^2 + \frac{128\pi}{r} \rightarrow S' = 2\pi r - \frac{128\pi}{r^2} = \frac{2\pi r^3 - 128\pi}{r^2}$$

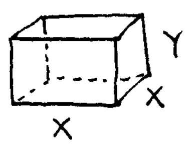
$$= \frac{2\pi(r^3 - 64)}{r^2} = 0$$

$-$ 0 $+$ S'

$r=0$ $r = 4$ in. and $h = 4$ in. and

abs. min. $S = 48\pi$ in.^{2}

II.)



volume $X^2 Y = 80 \rightarrow Y = \frac{80}{X^2}$,

minimize cost

$$C = 5(X^2) + 2(4XY) = 5X^2 + 8X\left(\frac{80}{X^2}\right) = 5X^2 + \frac{640}{X} \rightarrow$$

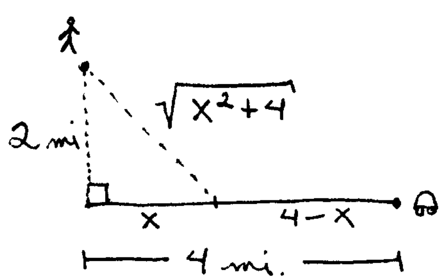
$$C' = 10X - \frac{640}{X^2} = \frac{10X^3 - 640}{X^2} = \frac{10(X^3 - 64)}{X^2} = 0$$

$-$ 0 $+$ C'

$X = 4$ ft., $Y = 5$ ft. and

abs. min. $C = \$240$

III.)



woods: 3 mph

road: 5 mph

$T = \frac{D}{R}$

minimize time

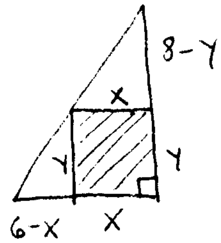
$$T = \frac{\sqrt{X^2+4}}{3} + \frac{4-X}{5} \rightarrow T' = \frac{1}{3}\cdot\frac{1}{2}(X^2+4)^{-\frac{1}{2}}\cdot 2X - \frac{1}{5}$$

$$= \frac{X}{3\sqrt{X^2+4}} - \frac{1}{5} = 0 \rightarrow 5X = 3\sqrt{X^2+4} \longrightarrow$$

$$25X^2 = 9(X^2+4) \rightarrow 16X^2 = 36 \rightarrow X = 3/2$$

$-$ 0 $+$ T'

$X = 3/2$ mi. and abs. min. $T = \frac{4}{3}$ hr.

IV.) A.) 

By similar Δ's

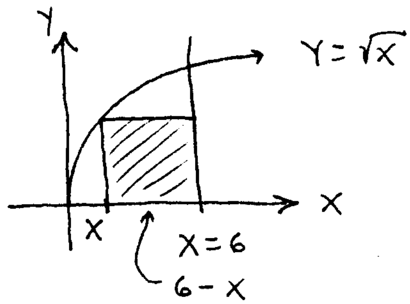
$$\frac{8}{6} = \frac{Y}{6-X} \rightarrow 48 - 8X = 6Y$$
$$\rightarrow Y = 8 - \tfrac{4}{3}X$$

maximize area $A = XY = X\left(8 - \frac{4}{3}X\right) = 8X - \frac{4}{3}X^2 \rightarrow$

$$A' = 8 - \frac{8}{3}X = 0 \longrightarrow X = 3$$

$$\underset{X=3}{\overset{+ \quad 0 \quad -}{\longrightarrow}} \quad A'$$

$Y = 4$ and abs. max. $A = 12$.

B.) 

maximize area

$$A = (6-X)Y$$
$$= (6-X)\sqrt{X} = 6\sqrt{X} - X^{3/2} \rightarrow$$

$$A' = 6\frac{1}{2\sqrt{X}} - \frac{3}{2}\sqrt{X} = \frac{3}{\sqrt{X}} - \frac{3\sqrt{X}}{2}$$

$$\rightarrow A' = \frac{6-3X}{2\sqrt{X}} = 0 \rightarrow X = 2$$

$$\underset{X=2}{\overset{+ \quad 0 \quad -}{\longrightarrow}} \quad A'$$

so dimensions are 4 by $\sqrt{2}$ and abs. max. $A = 4\sqrt{2}$.

V.) Let X be the number of \$5 increases, then maximize revenue

$$R = (\#\text{ of rooms})(\text{charge per room})$$
$$= (100 - 4X)(50 + 5X)$$
$$= 5000 + 300X - 20X^2 \xrightarrow{D}$$
$$R' = 300 - 40X = 0 \longrightarrow X = \frac{30\cancel{0}}{4\cancel{0}} = 7\tfrac{1}{2}$$

$$\underset{X=7\frac{1}{2}}{\overset{+ \quad 0 \quad -}{\longrightarrow}} \quad R'$$

$X = 7\frac{1}{2}$ \$5 increases

max revenue $R = \$6125$,

\# rooms $= 70$, charge per room $= \$87.50$

VI. x: demand $\quad p$: price

$x = \dfrac{c}{p^2}$ and $p = \$20$, $x = 125$ boxes so

$125 = \dfrac{c}{400} \rightarrow c = 50{,}000$ so

$x = \dfrac{50{,}000}{p^2}$ or price $p = \sqrt{\dfrac{50{,}000}{x}}$;

cost $C = 750 + 5x$ so profit

$P_r = (\text{revenue}) - (\text{cost})$

$= px - (750 + 5x)$

$= \sqrt{\dfrac{50{,}000}{x}} \cdot x - 750 - 5x = \sqrt{50{,}000} \cdot \sqrt{x} - 750 - 5x$;

$P_r' = \sqrt{50{,}000} \cdot \dfrac{1}{2\sqrt{x}} - 5 = 0 \rightarrow \cdots \rightarrow x = 500$ boxes

$+ \quad 0 \quad -$ $\;p'$ at $x = 500$ boxes, $p = \$10$ and max. profit is

$P_r = \$1750$