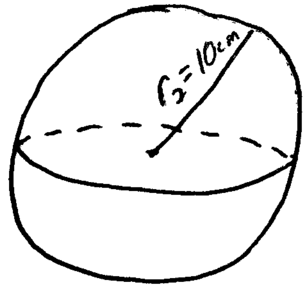
Section 1.3

39)

$r_1 = 1cm$

$V_1 = \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi \, cm^3$



$V_2 = \frac{4}{3}\pi r_2^3$

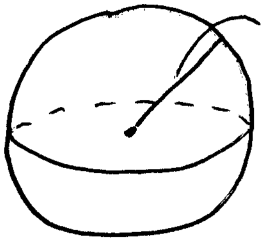
$= \frac{4}{3}\pi \, 10^3 cm^3$

$\frac{r_2}{r_1} = 10^1 \Rightarrow$ The radius of the larger ball is 1 order of magnitude greater.

$\frac{V_2}{V_1} = \frac{\frac{4}{3}\pi 10^3}{\frac{4}{3}\pi} = 10^3 \Rightarrow$ The volume of the larger ball is 3 orders of magnitude greater.

41) Typical Bacterium

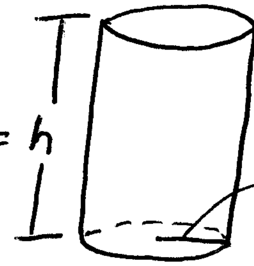
$r_1 = \frac{1}{2} - 1\mu m$



$V_{min} = \frac{4}{3}\pi \left(\frac{1}{2}\right)^3 \mu m^3 = \frac{1}{6}\pi \mu m^3$

$V_{max} = \frac{4}{3}\pi (1)^3 \mu m^3 = \frac{4}{3}\pi \mu m^3$

Epulopiscium Fishelsoni



$600\mu m = h$

$r_2 = 80\mu m$

$V_2 = \pi r_2^2 h = \pi (80)^2 600 \mu m^3$

$= 3.84 \times 10^6 \pi \mu m$

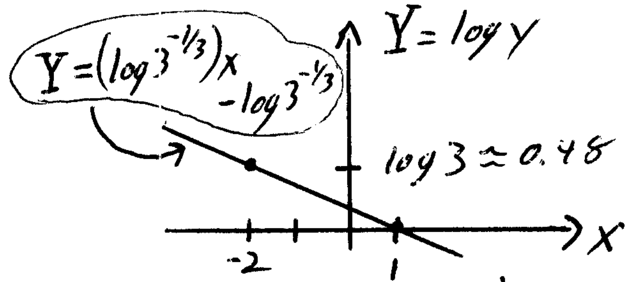
Min Volume Ratio: $\frac{V_2}{V_{max}} = \frac{3.84 \times 10^6 \pi}{\frac{1}{6}\pi} = 2.304 \times 10^7$

Max Volume Ratio: $\frac{V_2}{V_{min}} = \frac{3.84 \times 10^6 \pi}{4/3 \, \pi} = 2.88 \times 10^6$

Then, the volume of Epulopiscium Fishelsoni is 6-7 orders of magnitude greater.

45)

x	y	$Y = \log y$
-2	3	0.48
1	1	0



$$m = \frac{Y_1 - Y_2}{x_1 - x_2} = \frac{\log 3 - 0}{-2-1} = -\frac{1}{3}\log 3 = \log 3^{-1/3}$$

$$Y - 0 = \log 3^{-1/3}(x-1) = (\log 3^{-1/3})x - \log 3^{-1/3}$$

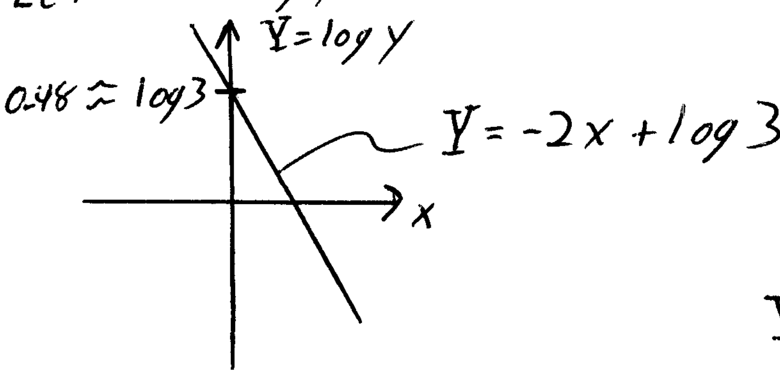
$$\Rightarrow \log y = (\log 3^{-1/3})x - \log 3^{-1/3} = \log (3^{-1/3})^x + \log 3^{1/3}$$

$$\Rightarrow \log y = \log\left[3^{1/3}(3^{-1/3})^x\right] \Rightarrow \boxed{y = 3^{1/3}(3^{-1/3})^x}$$

47) $y = 3\cdot 10^{-2x} \Rightarrow \log y = \log 3\cdot 10^{-2x} = \log 3 + \log 10^{-2x}$

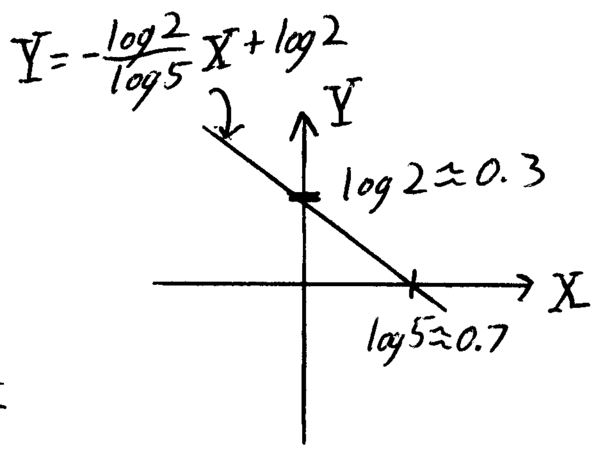
$\Rightarrow \log y = \log 10^{-2x} + \log 3 = -2x\log 10 + \log 3 = -2x + \log 3$

Let $Y = \log y$. Then $\boxed{Y = -2x + \log 3}$



55)

x	$X = \log x$	y	$Y = \log y$
1	0	2	0.3
5	0.7	1	0



$$m = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{\log 2 - 0}{0 - \log 5} = -\frac{\log 2}{\log 5}$$

$$\Rightarrow Y = -\frac{\log 2}{\log 5}X + \log 2 \Rightarrow \log y = -\frac{\log 2}{\log 5}\log x + \log 2$$

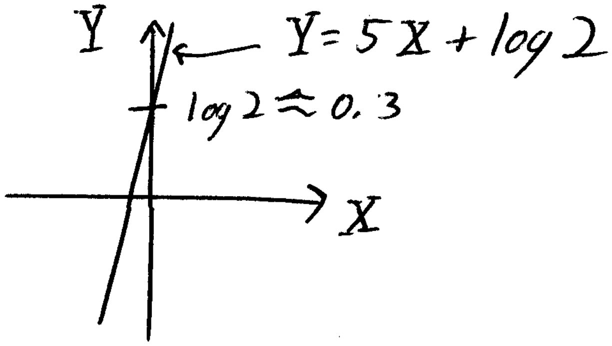
$$\Rightarrow \log y = \log x^{-\log 2/\log 5} + \log 2 = \log 2\cdot x^{-\log 2/\log 5}$$

$$\Rightarrow \boxed{y = 2x^{-\log 2/\log 5}}$$

59) $y=2x^5 \Rightarrow \log y = \log 2x^5 = \log 2 + \log x^5$

$\Rightarrow \log y = \log x^5 + \log 2 = 5\log x + \log 2$

Let $Y = \log y$ and $X = \log x$. Then, $\boxed{Y = 5X + \log 2}$



68) $g(s) = 1.8e^{-0.2s} \Rightarrow \log g = \log 1.8e^{-0.2s} = \log 1.8 + \log e^{-0.2s}$

$\Rightarrow \log g = \log e^{-0.2s} + \log 1.8 = -0.2s \log e + \log 1.8$

$\Rightarrow \log g = (-0.2\log e)s + \log 1.8$

Use semi-log (aka. log-linear) plot

69) $N(t) = 130 \cdot 2^{1.2t} \Rightarrow \log N = \log 130 \cdot 2^{1.2t} = \log 130 + \log 2^{1.2t}$

$\Rightarrow \log N = \log 2^{1.2t} + \log 130 = 1.2t \log 2 + \log 130$

$\Rightarrow \log N = (1.2\log 2)t + \log 130$

Use semi-log (a.k.a. log-linear) plot

71) $R(t) = 3.6t^{1.2} \Rightarrow \log R = \log(3.6t^{1.2}) = \log 3.6 + \log t^{1.2}$

$\Rightarrow \log R = \log t^{1.2} + \log 3.6 = 1.2\log t + \log 3.6$

$\Rightarrow \log R = 1.2 \log t + \log 3.6$.

Use log-log plot

72) $L(c) = 1.7 \cdot 10^{2.3c} \Rightarrow \log L = \log(1.7 \cdot 10^{2.3c}) = \log 1.7 + \log 10^{2.3c}$

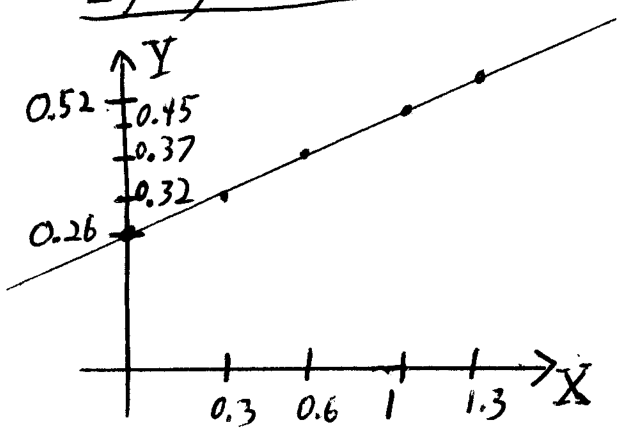
$\Rightarrow \log L = \log 10^{2.3c} + \log 1.7 = 2.3c + \log 1.7$

$\Rightarrow \log L = 2.3c + \log 1.7$. Use log-log plot.

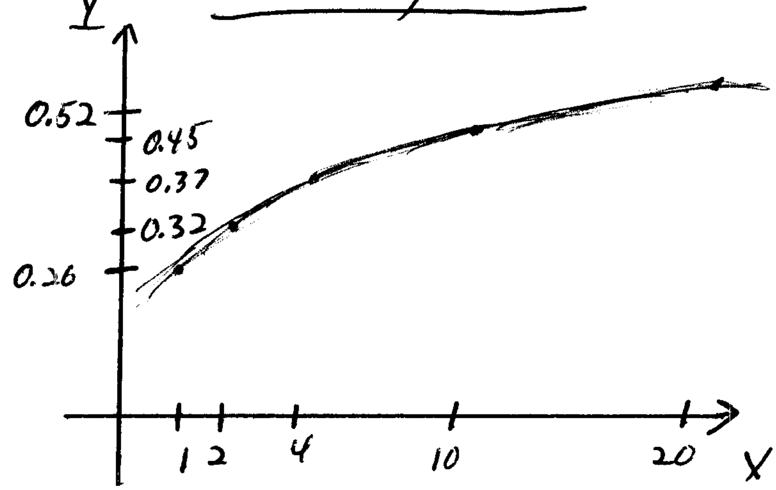
73)

x	$X = \log x$	y	$Y = \log y$
1	0	1.8	0.26
2	0.30	2.07	0.32
4	0.60	2.38	0.37
10	1	2.85	0.45
20	1.30	3.28	0.52

Log-log Plot



Semi-Log Plot



The Log-log plot looks more like a straight line, so using points $(0, 0.26)$ and $(1, 0.45)$, we get

$$m = \frac{Y_1 - Y_2}{X_1 - X_2} = \frac{0.45 - 0.26}{1 - 0} = 0.19 \; ; \; Y\text{-int} = b = 0.26 = \log 1.8$$

$$\Rightarrow Y = mX + b = 0.19X + \log 1.8 \Rightarrow \log y = 0.19 \log x + \log 1.8$$

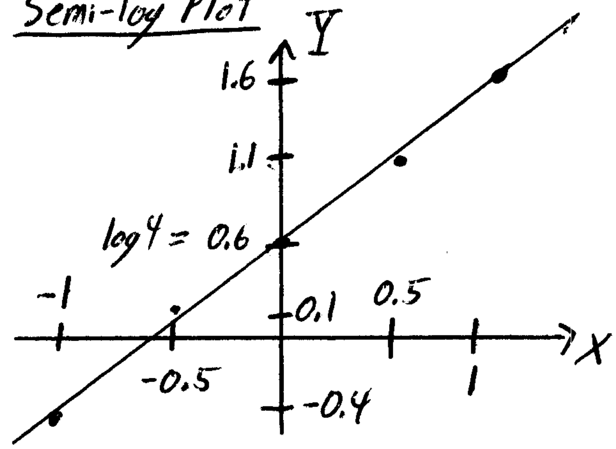
$$\Rightarrow \log y = \log x^{0.19} + \log 1.8 = \log(1.8x^{0.19})$$

$$\Rightarrow \boxed{y = 1.8x^{0.19}}$$

75)

x	$X = \log x$	y	$Y = \log y$
-1	DNE	0.398	-0.4
-0.5	DNE	1.26	0.1
0	DNE	4	0.6
0.5	-0.3	12.68	1.1
1	0	40.18	1.6

Semi-log Plot



Clearly, since X D.N.E. for certain x-values, we need to use the Semi-log plot.

So we use points $(1, 1.6)$ and $(0, 0.6)$ and get

$$m = \frac{Y_1 - Y_2}{x_1 - x_2} = \frac{1.6 - 0.6}{1 - 0} = 1 = \log 10 \text{ ; } Y\text{-int} = b = \log 4$$

$$\Rightarrow Y = mx + b = (\log 10)x + \log 4 = \log 10^x + \log 4 = \log(4 \cdot 10^x)$$

$$\Rightarrow \log y = \log(4 \cdot 10^x) \Rightarrow \boxed{y = 4 \cdot 10^x}$$

83) This will be an optional assignment.